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On-line reactivity calculation using Lagrange method

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ABSTRACT

In this paper, a novel multi-step method is proposed for solving the inverse point kinetics problem using Lagrange polynomial method. By use of this approach, the need for nuclear power history or the Laplace transform is vanished. Furthermore, the accuracy of the method is of order h^n for the $(n+1)$-point formula, where h is the computational time-step.

The three- and five-point formulas of the Lagrange method are used for on-line reactivity calculations and results are benchmarked against reference solutions for different nuclear power forms. Moreover, results for different computational time-steps are compared in each case.

The results show the accuracy of the proposed method in all benchmarking cases. For slow transients (large reactor periods), it is shown that time-steps of up to 1 s lead to highly reliable reactivity calculations. However, the optimal time-step in almost all cases is shown to be 0.1 s. The main advantage of the proposed approach, in contrast with previous numerical methods, is its stability and convergence in large time-step calculations.

The proposed method can be used as real time reactivity meter in all nuclear reactors without limitation of nuclear power form.

1. Introduction

The inverse solution of the point kinetic equations, in order to obtain the reactivity $\rho(t)$ where the variations of the reactor power $P(t)$ are known arbitrary functions of time, is of great importance for several reasons: (a) In reactor operation, the time dependence of the applied reactivity required to yield a specific power variation must be known in order to program the control rod motion (Murray et al., 1964). (b) During the start-up of a nuclear reactor, the design of a continuous reactivity monitor is needed from the viewpoint of reactor safety (Suzuki and Tsunoda, 1964). Furthermore, there are some other applications for the inverse kinetics problem (Akcasu et al., 1971; Hetrick, 1971).

In the literature, there have been some researches on calculation of the reactivity in a nuclear reactor based on the numerical solution of the integral term associated with the inverse point kinetics equation, known as the nuclear power history (Shimazu et al., 1987; Hoogenboom and Van Der Sluijs, 1988; Binney and Bakir, 1989; Ansari, 1991; Tamura, 2003). However, in order to design an on-line reactivity meter for a nuclear reactor, the need for the power history must be vanished. For this aim, a method was introduced to calculate the reactivity without using the nuclear power history (Suescún et al., 2007), implementing nuclear power derivatives and the least squares method to control the noise. Reactivity can also be calculated through finite differences (Suescún and Senra, 2010). Recently, a new formulation has also been presented to calculate the reactivity in nuclear reactors for different forms of nuclear power using the Hamming method (Suescún et al., 2012).

In this work, Lagrange difference formulas are proposed and tested for the on-line reactivity calculation without the need for nuclear power history or the Laplace transform. The truncation error accompanied by each formula is analyzed and results for different forms of the nuclear power are benchmarked against reference solutions. Furthermore, results are compared with some other existing methods in the literature.

In the following and in Section 2, the inverse point kinetics problem is explained in detail. Section 3 describes the Lagrange method and its implementation for on-line reactivity calculations. The results of the proposed method for different nuclear power forms are shown then in Section 4. Finally, Section 5 concludes the paper.

2. Inverse point kinetics problem

The point kinetic equations can be obtained from the time-dependent neutron diffusion equation, assuming constant shape of the neutron flux (Duderstadt and Hamilton, 1976). Supposing six delayed-neutron groups, the system of equations comprises seven non-linear coupled differential equations, which describe the time evolution of the neutron density (or the nuclear power) and concentration of delayed-neutron precursors as follows:

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$$\frac{dP(t)}{dt} = \left[\frac{\rho(t)-\beta}{\Lambda}\right]P(t) + \sum_{i=1}^{6}\lambda_i C_i(t) \tag{1}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda}P(t) - \lambda_i C_i(t); \quad i = 1,2,\ldots,6 \tag{2}$$

With the following initial conditions:

$$P(0) = P_0 \tag{3}$$

$$C_i(0) = \frac{\beta_i}{\Lambda\lambda_i}P_0, \tag{4}$$

where $P(t)$ is the neutron number density (proportional to nuclear power), $C_i(t)$ the concentration of the ith group of delayed-neutron precursors, $\rho(t)$ the reactivity Λ the neutron generation time, β_i the effective fraction of the ith group of delayed neutrons, β the total effective fraction of delayed neutrons ($\beta = \sum_i \beta_i$), λ_i is the decay constant of the ith group of delayed-neutron precursors.

Many formulas have been derived for calculating the reactivity $\rho(t)$ from a prescribed power history $P(t)$ (Duderstadt and Hamilton, 1976; Hetrick, 1971). From Eqs. (1) and (2) one has

$$\rho(t) = \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{\Lambda}{P(t)}\sum_{i=1}^{6}\lambda_i C_i(t) \tag{5}$$

$$\frac{dC_i(t)}{dt} + \lambda_i C_i(t) = \frac{\beta_i}{\Lambda}P(t); \quad i = 1,2,\ldots,6 \tag{6}$$

Integrating Eq. (6), the precursor densities can be stated as follows (Hetrick, 1971):

$$C_i(t) = C_i(0)e^{-\lambda_i t} + \frac{\beta_i}{\Lambda}\int_0^t e^{-\lambda_i(t-t')}P(t')dt' \tag{7}$$

This equation is obtained supposing that $P(t \leqslant 0) = P_0$ and $C_i(t \leqslant 0) = C_i(0)$, which means the reactor is in critical state. Substituting Eq. (7) into Eq. (5), after some manipulations, gives

$$\rho(t) = \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{1}{P(t)}\sum_{i=1}^{6}\lambda_i\beta_i e^{-\lambda_i t}\left[\frac{P_0}{\lambda_i} + \int_0^t e^{\lambda_i t'}P(t')dt'\right] \tag{8}$$

This equation cannot be applied as on-line reactivity meter because of its dependence on the nuclear power history (the integral in the RHS of this equation). However, it can be solved analytically for some nuclear power forms. Hence, it will be considered in the following as the reference solution for benchmarking numerical methods. For instance, if the power is assumed to be as $P(t) = \exp(\omega t)$, the analytical solution of Eq. (8) will be as follows:

$$\rho(t) = \beta + \Lambda\omega - \sum_{i=1}^{6}\frac{\lambda_i\beta_i}{(\lambda_i+\omega)} - \sum_{i=1}^{6}\frac{\beta_i\omega}{(\lambda_i+\omega)}e^{-(\lambda_i+\omega)t}, \tag{9}$$

which results in the Inhour equation as $t \to \infty$. The Inhour equation (Hetrick, 1971), nevertheless, is obtained assuming a constant reactivity $\rho(t) = \rho_0$. Furthermore, analytical solutions of the integral in the RHS of Eq. (8) for some other power variations are given in Table 1.

Table 1
Analytical solution of the integral term in the inverse kinetics equation for some power changes.

$n(t')$	$\int_0^t e^{\lambda_i t'}n(t')dt'$
$e^{\omega t'}$	$\frac{e^{(\lambda_i+\omega)t}-1}{\lambda_i+\omega}$
$1+\omega t'$	$\frac{1}{\lambda_i^2}[(\lambda_i-\omega)(e^{\lambda_i t}-1)+\omega\lambda_i t e^{\lambda_i t}]$
$1+A\sin\omega t'$	$\frac{1}{\lambda_i}(e^{\lambda_i t}-1)+\frac{A}{\lambda_i^2+\omega^2}[e^{\lambda_i t}(\lambda_i\sin\omega t-\omega\cos\omega t)+\omega]$

3. Lagrange method

3.1. Lagrange difference formulas

To obtain general derivative approximation formulas, suppose that $\{t_0, t_1, \ldots, t_n\}$ are $(n+1)$ distinct numbers in some interval I and that $f \in C^{n+1}(I)$. The Lagrange polynomial expansion of $f(t)$ can be expressed as follows (Burden and Faires, 2010):

$$f(t) = \sum_{k=0}^{n} f(t_k)L_k(t) + \frac{(t-t_0)\ldots(t-t_n)}{(n+1)!}f^{(n+1)}(\xi(t)) \tag{10}$$

for some $\xi(t)$ in I, where $L_k(t)$ denotes the kth Lagrange coefficient polynomial for $f(t)$ at $t_0, t_1, \ldots, t_n$.

Differentiating this expression gives:

$$f'(t) = \sum_{k=0}^{n} f(t_k)L_k'(t) + D_t\left[\frac{(t-t_0)\ldots(t-t_n)}{(n+1)!}\right]f^{(n+1)}(\xi(t)) + \frac{(t-t_0)\ldots(t-t_n)}{(n+1)!}D_t\left[f^{(n+1)}(\xi(t))\right] \tag{11}$$

One confronts a problem estimating the truncation error in Eq. (11) unless t is one of the numbers t_j. In this case, the term multiplying $D_t[f^{(n+1)}(\xi(t))]$ is zero, and the formula becomes:

$$f'(t_j) = \sum_{k=0}^{n} f(t_k)L_k'(t_j) + \frac{f^{(n+1)}(\xi(t_j))}{(n+1)!}\prod_{\substack{k=0\\k\neq j}}^{n}(t_j - t_k), \tag{12}$$

which is called a $(n+1)$ -point formula to approximate $f'(t_j)$.

Generally, using more evaluation points in Eq. (12) results in more accuracy. However, growth of round-off error due to the number of functional evaluations should be also considered. Besides, the method would produce more delay if more evaluation points are considered and real time calculations would be infeasible if calculation time is increased. Thus, the most common formulas are three- and five-point ones (Burden and Faires, 2010).

Three-point endpoint formula is given as:

$$f'(t_j) = \frac{1}{2h}[3f(t_j) - 4f(t_j - h) + f(t_j - 2h)] + \frac{h^2}{3}f^{(3)}(\xi), \tag{13}$$

where ξ lies between t_j and $t_j - 2h$. In addition, five-point endpoint formula is obtained as follows:

$$f'(t_j) = \frac{1}{12h}[25f(t_j) - 48f(t_j - h) + 36f(t_j - 2h) - 16f(t_j - 3h) + 3f(t_j - 4h)] + \frac{h^4}{5}f^{(5)}(\xi), \tag{14}$$

in which ξ lies between t_j and $t_j - 4h$. Furthermore, h denotes the computational time-step in both the formulas.

According to Eqs. (13) and (14), the truncation errors accompanied by the three- and five-point formulas are of order h^2 and h^4, respectively.

3.2. Reactivity calculation using Lagrange method

Introducing the three-point formula (Eq. (13)) into Eqs. (5) and (6) gives the following equations for the reactivity calculation:

$$\rho^{n+1} = \beta + \frac{\Lambda}{P^{n+1}}\frac{1}{2h}(3P^{n+1} - 4P^n + P^{n-1}) - \frac{\Lambda}{P^{n+1}}\sum_i \lambda_i C_i^{n+1} \tag{15}$$

$$C_i^{n+1} = \left(\frac{3}{2h} + \lambda_i\right)^{-1}\left[\frac{\beta_i}{\Lambda}P^{n+1} + \frac{1}{2h}(4C_i^n - C_i^{n-1})\right] \tag{16}$$