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Spatial recondensation using the Discrete Generalized Multigroup method

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ABSTRACT

Cross-section recondensation using the Discrete Generalized Multigroup method (DGM) has shown promise in improving coarse group solutions by capturing neighboring spectral effects. However, full consistency with the fine group is only assured when using a spatially flat angular flux approximation, such as used in step difference discrete ordinates. Moving to high order spatial methods, such as characteristic type methods, reveals spatial inconsistencies that exist between the DGM equations and the original fine group equations. We propose two methods to address the spatial inconsistencies between DGM and the fine group equations. The first method introduces local spatial dependence of the angular and scalar fluxes, determined using higher order spatial methods, into the cross section moments and del terms defined in DGM. This provides much better agreement between the DGM solution and the fine group solution for any high order spatial method. Unfortunately, this process introduces significant increases to the required memory storage. This issue can be mitigated to some extent through replacement of storage with on-the-fly calculations and a procedure to do so is outlined as well. The second method defines an exact del term by applying the spatial method to the fine group equations before deriving the DGM equations. This new definition allows for convergence to the exact fine group solution. While these equations need to be built specifically for a given spatial method and still suffer from large storage requirements, higher order spatial methods are no longer required and introducing physics informed approximations should significantly reduce the memory burden.

1. Introduction

The multigroup approximation of the neutron transport equations is the discretization of choice in nuclear reactor analysis. For whole core analysis, the multigroup approximation is incorporated into a multistep process, in which the energy resolution is gradually lowered to allow for more spatial and angular unknowns to be introduced into the core model. The approximations used in this process prevent fine group information from being passed between neighboring materials with significantly different properties. Historically, this approximation has not presented LWR analysis with problems since the material properties, for the most part, do not change significantly between assemblies and the neutron mean free paths are much smaller than the size of the assemblies. Many advanced reactor cores, though, may contain assemblies that are much different from one another, such as MOX loaded cores, and may have long neutron mean free paths, like High Temperature Gas Reactors. In these cases, the multigroup approximation may lead to significant errors in important reactions rates. Therefore, a new approach was developed to regain fine group accuracy at the core level without the computational cost of a full fine group solve.

The Discrete Generalized Multigroup (DGM) provides a way to reconstruct fine group fluxes using only the solution to the coarse group equations and high order in energy fixed source equations. Still, this too is limited by the approximations used in multistep process, since we assume the fine group spectrum at the assembly level is comparable to what it will be when placed in a reactor. To correct these errors, the reconstructed fine group fluxes can be used to update coarse group constants and take into account the spectral changes between different assembly types. This process is called recondensation. By successively solving the coarse group equations using updated coarse group constants, the coarse group solution can be improved and approach the fine group solution. This provides a way to achieve near fine group accuracy at a cost comparable to a coarse group calculation (Zhu and Forget, 2010, 2011).

While DGM works perfectly for flat flux methods, such as step difference discrete ordinates, The DGM equations are not fully consistent with the fine group equations when applied to higher order spatial methods. This limitation to flat flux methods requires a high spatial resolution to provide an accurate solution. Enabling

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consistency with higher order spatial methods would reduce the total number of spatial unknowns solved and allow recondensation to be applied to a wider range of methods for whole core analysis.

This paper proposes two different approaches to addressing this issue. The first builds upon the current form of the DGM equations and its respective cross section moment definitions. Section 2 provides the foundation of DGM and the recondensation procedure necessary to begin building a consistent approach. Section 3 introduces the source of spatial inconsistencies in DGM. Section 4 presents the first proposed approach to deal with high order spatial methods, while Section 5 uses this approach to improve the coarse group solution. Section 6 defines a new set of DGM equations derived specifically for MOC that allows recondensation to converge to the exact fine group solution without the need of additional higher spatial order equations.

2. Recondensation using the Discrete Generalized Multigroup method

2.1. The Discrete Generalized Multigroup (DGM) method

Using the multigroup approximation in the group collapse process, it is assumed that the fine group spectrum within each coarse group is flat. In doing so, all information about the fine group spectrum is lost. With DGM, it is assumed that the spectrum within each coarse group is expanded using a set of orthogonal functions. While continuous functions in energy can be used to represent the fine group spectrum (Rahnema et al., 2008), discrete basis functions are a more natural fit for the discrete nature of the multigroup equations. This representation of the within-group fluxes provides the basis for the formation of the DGM moment equations. Through the substitution of this expansion into the fine group equation and the subsequent integration with respect to the ith discrete orthogonal function, the isotropic DGM moment equations are produced (Zhu and Forget, 2010).

$$\vec{\Omega}\cdot\nabla\psi_{i,g}\left(\vec{r},\vec{\Omega}\right)+\Sigma_{T,0,g}\left(\vec{r},\vec{\Omega}\right)\psi_{i,g}\left(\vec{r},\vec{\Omega}\right)+\delta_{i,g}\left(\vec{r},\vec{\Omega}\right)\psi_{0,g}\left(\vec{r},\vec{\Omega}\right)$$
$$=\frac{1}{4\pi}\sum_{g'=1}^{G}\Sigma_{s,i,g'\to g}(\vec{r})\phi_{0,g'}(\vec{r})+\frac{\chi_{i,g}(\vec{r})}{4\pi k}\sum_{g'=1}^{G}\nu\Sigma_{f,g'}(\vec{r})\phi_{0,g'}(\vec{r})$$

where

$$\nu\Sigma_{f,g}(\vec{r})=\sum_{k\in g}P_0(K,N)\phi_g(\vec{r},K)\nu\Sigma_{f,g}(\vec{r},K)\Big/\sum_{k\in g}P_0(K,N)\phi_g(\vec{r},K)$$

$$\Sigma_{s,i,g'\to g}(\vec{r})=\sum_{L\in g'}\sum_{k\in g}P_i(K,N)\phi_{g'}(\vec{r},L)\Sigma_s(\vec{r},L\to K)\Big/\sum_{L\in g'}P_0(L,M)\phi_{g'}(\vec{r},L)$$

$$\Sigma_{T,0,g}(\vec{r},\vec{\Omega})=\sum_{k\in g}P_0(K,N)\psi_g(\vec{r},\vec{\Omega},K)\Sigma_{T,g}(\vec{r},K)\Big/\sum_{k\in g}P_0(K,N)\psi_g(\vec{r},\vec{\Omega},K)$$

$$\delta_{i,g}(\vec{r},\vec{\Omega})=\sum_{k\in g}P_i(K,N)\psi_g(\vec{r},\vec{\Omega},K)(\Sigma_{T,g}(\vec{r},K)-\Sigma_{T,0,g}(\vec{r},\vec{\Omega}))\Big/\sum_{k\in g}P_0(K,N)\psi_g(\vec{r},\vec{\Omega},K)$$

At this point, we still have the same number of equations as the original fine group problem, but what has changed is how we use these equations. With DGM, we find that we only have to conduct our power iteration on our 0th order DGM equation, which is equivalent to the coarse group problem since the del term vanishes for $i = 0$. Therefore, instead of having to conduct a power iteration over all the fine group equations, we only have to do so on the coarse group equations.

$$L_{0,g}\psi_{0,g}=Q_{0,g}(k,\phi_{0,g})$$

Since the other DGM equations only depend on the solution of the 0th order DGM equation, once we have converged on our coarse group solution, all that is left to do is solve a set of fixed source problems to calculate the rest of our angular flux moments.

$$L_{0,g}\psi_{i,g}=Q_{i,g}(k,\psi_{0,g},\phi_{0,g})$$

If we have the correct cross section moments for our problem, then we can solve the DGM moment equations for our angular flux moments and reconstruct the exact fine group solution.

$$\psi_g(\vec{r},\vec{\Omega},K)=\sum_{i=0}^{N-1}a_iP_i(K,N)\psi_{i,g}\left(\vec{r},\vec{\Omega}\right)$$

2.2. Recondensation

Unfortunately, the correct calculation of our cross section moments assumes a priori knowledge of the true fine group solution. Since we gain nothing from applying the DGM equations if the fine group eigenvalue problem has already been solved, another method is required. This led to the development of a process called recondensation (see Fig 1).

To initialize the recondensation process, we assume an initial guess on our fine group flux and calculate our cross section moments accordingly. With these values, we conduct our power iteration on the coarse group equations, this solution is used to solve the higher order DGM equations and a new set of fine group fluxes is calculated. The new step that is introduced is that we use these new flux values in the group collapse process to obtain a new set of cross section moments. This is the essence of recondensation.

These updated material properties are used in the next DGM calculation, the fine group fluxes reconstructed and the process is repeated. After a number of recondensation steps have occurred, the reconstructed fine group fluxes will converge towards the true fine group solution (Zhu and Forget, 2011).

This method initially suffered from stability issues leading to divergence in many cases. This has recently been resolved by application of a Krasnoselskij Iteration on the updated angular flux moments instead of the typical Picard Iteration. This modifies the recondensation process as follows:

$$\psi_{i,g}^{k+1}=(1-\lambda)\psi_{i,g}^{k}+\lambda T\psi_{i,g}^{k}$$

where T denotes the original recondensation process in operator form, k is the current iteration and lambda may vary from 0 to 1.

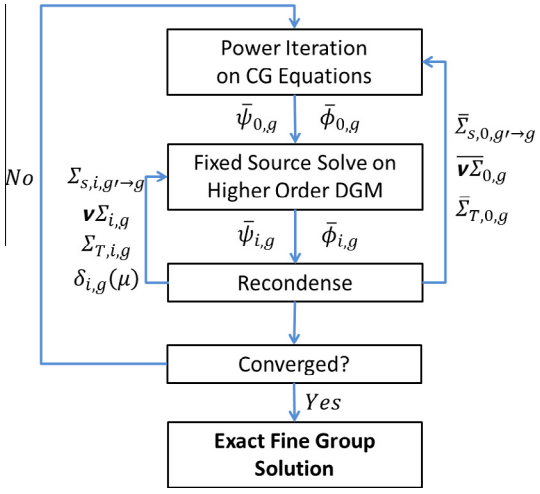

Fig. 1. Flowchart for the recondensation procedure.