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Technical note

Calculation of reactivity in subcritical reactors using the method of partial derivatives

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ABSTRACT

The analysis of source-driven subcritical reactors is of great interest as it allows savings in the mineral resources used in their cores, due to the possible transmutation of nuclear fuel burned in critical reactors. In ADS (Accelerator Driven System) reactors the neutrons produced from spallation reactions generate an source, external to the core, that supports its operation. Although there are different formalisms to describe the kinetic behaviour of these reactors, its functioning remains an open question, as there is no ADS reactor in operation. Thus, with the aim of predicting reactivity behaviour in this type of reactor, the inverse point kinetics equation was obtained, using a specific formalism for subcritical reactors. The results obtained were compared to Monte Carlo simulations for the purposes of validation and were shown to be coherent, displaying deviations under 100 pcm.

1. Introduction

Source-driven subcritical reactors have been considered around the world as an efficient way for the transmutation of large amount of radioactive waste, which helps reducing the volumes in geological storage (Gandini, 2002). Apart from this, they may be an important power generation alternative to the use of critical reactors, which present more safety-related problems (Piera et al., 2010). These reactors, also called hybrid reactors, are still in development, which includes the studying of new fuels and reprocessing techniques.

The kinetic behaviour of these reactors is described by different point kinetics equations systems (Gandini, 2000; Nishihara et al., 2003; Silva et al., 2012). Thus, aiming at predicting the reactivity behaviour in subcritical reactors, an expression of inverse point kinetics was obtained. The monitoring of the reactivity is critical to the operation of any nuclear reactor and becomes even more important in reactors operating in subcriticality, requiring a faster method of calculation for this type of reactors. The point kinetics model is also used for transient analyses and for the simulation of operation transients (Rineiski et al., 2005).

The validation of the proposed inverse point kinetics for subcritical systems was done through the Monte Carlo (MC) simulation.

2. Point kinetics equations

The formulation of the point kinetic equations proved to be a powerful tool for the evaluation of reactivity for various types of reactors (Dulla et al., 2003). This formulation describes the behaviour of neutrons within the reactor with no spatial dependence, and the distribution of neutrons develops in a punctual manner.

The different representations found in the literature for the importance function in ADS reactors results in different set of equations. These equations satisfactorily describe subcritical systems (Silva et al., 2012) and can be obtained from the transport equation or from the diffusion equation that govern the neutrons. The space profile of the neutron flux in the subcritical assembly is strongly influenced by both the neutron multiplication factor and external neutron source (Iwanaga et al., 2008). This influence leads to new mathematical considerations for the description of the physical system (Dulla et al., 2003) due to the external neutron source.

In this paper the importance function as proposed by Silva et al. (2012) will be considered. This importance function presents an adjoining source term that becomes significant for systems that are far from criticality. This term is a function of the multiplication factor $0.95 < k_{eff} < 1$ and implies the following point kinetics equation system for a subcritical reactor:

$$\Lambda\frac{dP(t)}{dt} = (\rho(t)-\beta)P(t) + \lambda\varpi\Sigma_f C(t) + P(t)\Gamma + \varpi\Sigma_f q \quad (1)$$

$$\frac{dC(t)}{dt} = \frac{\beta}{\varpi\Sigma_f}P(t) - \lambda C(t), \quad (2)$$

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where P is the nuclear power, β is the delayed neutron fraction, $\rho(t)$ is the reactivity of the system, $C(t)$ is the concentration of precursors, Λ is the mean neutron generation time, ϖ is the average energy released by fission, Γ is a normalizing factor, and Σ_f is the fission macroscopic cross-section.

3. Inverse point kinetics

For the case of subcritical reactors, reactivity monitoring is important during reactor startup, for low-power physical tests and during full-power operation. This is due to the fact that, as the nuclear fuel is burned one has to adjust the proton beam intensity to keep the production of electricity constant during normal operation.

The expression for reactivity can be easily obtained from the point kinetics equations which describe the evolution in time of the distribution of neutrons and of the concentrations of the delayed neutron precursors in the core of a nuclear reactor (Diaz et al., 2007).

To do this is necessary solve the Eqs. (1) and (2) to explicit $\rho(t)$. Thus, starting from Eq. (2) and using the integrating factor method, one has that:

$$C(t) = \frac{\beta}{\omega \cdot \Sigma_f}\int_{-\infty}^{t} P(t')e^{-\lambda(t'-t)}dt' \tag{3}$$

Replacing Eq. (3) in Eq. (1), one obtains that:

$$\frac{dP(t)}{dt} = (\rho(t)-\beta)\frac{P(t)}{\Lambda} + \frac{\lambda\beta}{\Lambda}\int_{-\infty}^{t} P(t')e^{-\lambda(t'-t)}dt' + \frac{\Gamma \cdot P(t)}{\Lambda} + Q, \tag{4}$$

where $Q \equiv \omega\Sigma_f q/\Lambda$ is the term for the external neutron source. Solving Eq. (4) for $\rho(t)$, one has:

$$\rho(t) = \beta + \Gamma + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{\lambda\beta}{P(t)}\int_{-\infty}^{t} e^{-\lambda(t-t')}P(t')dt' - \frac{\Lambda Q}{P(t)}. \tag{5}$$

Working out Eq. (5) we can find that,

$$\rho(t) = \beta + \Gamma + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{\beta P_0}{P(t)}e^{-\lambda t} - \frac{\Lambda Q}{P(t)} - \frac{\lambda\beta}{P(t)}I(t), \tag{6}$$

being $I(t)$ the nuclear power history,

$$I(t) = \int_0^t e^{-\lambda(t-t')}P(t')dt' \tag{7}$$

Integrating Eq. (7) by parts, it is possible to write

$$I(t) = \frac{P(t)}{\lambda} - \frac{P(0)\cdot e^{-\lambda\cdot t}}{\lambda} - \frac{1}{\lambda}\int_0^t P^{(1)}(t')\cdot e^{-\lambda\cdot(t-t')}dt', \tag{8}$$

where $P^{(1)}(t)$ is the first-order derivative of nuclear power in relation to the time.

Solving the integral in the Eq. (8) by parts again, a new expression that involves another integral with the derivative of second order of the nuclear power is obtained. So, repeating this procedure n times one obtains a series for the nuclear power with time dependence (Diaz et al., 2007). Thus, one can write that:

$$\int_0^t e^{-\lambda(t-t')}P(t')dt' + \int_0^t \frac{P^{(k+1)}(t')\cdot e^{-\lambda\cdot(t-t')}}{\lambda^{k+1}}dt' = \sum_{n=0}^{k}(-1)^n\frac{P^{(n)}(t)}{\lambda^{n+1}} - \sum_{n=0}^{k}(-1)^n\frac{P^{(n)}(0)\cdot e^{-\lambda\cdot t}}{\lambda^{n+1}} \tag{9}$$

Supposing k is an odd number, it follows that $k = 2k - 1$. Thus, Eq. (9) can be written as

$$\int_0^t e^{-\lambda(t-t')}P(t')dt' + \int_0^t \frac{P^{(2k)}(t')\cdot e^{-\lambda\cdot(t-t')}}{\lambda^{2k}}dt' = \sum_{n=0}^{2k-1}(-1)^n\frac{P^{(n)}(t)}{\lambda^{n+1}} - \sum_{n=0}^{2k-1}(-1)^n\frac{P^{(n)}(0)\cdot e^{-\lambda\cdot t}}{\lambda^{n+1}}. \tag{10}$$

Assuming the following conditions:

$$P^{(2n-1)}(t) = P^{(1)}(t)\left\{\frac{P^{(2)}(t)}{P(t)}\right\}^{n-1} \tag{11}$$

$$P^{(2n)}(t) = P^{(1)}(t)\left\{\frac{P^{(2)}(t)}{P(t)}\right\}^{n}, \tag{12}$$

being $\frac{P^{(2)}(t)}{P(t)} = c$, where c is a constant.Now one can write Eq. (10) as follows:

$$\int_0^t e^{-\lambda(t-t')}P(t')dt' = S_1 + S_2, \tag{13}$$

being:

$$S_1 = \frac{1}{\left[1-\left\{\frac{P^{(2)}(t)}{P(t)}\right\}^n\right]}\sum_{n=0}^{2k-1}\frac{(-1)^n}{\lambda^{n+1}}P^{(n)}(t) \tag{14}$$

$$S_2 = -\frac{1}{\left[1-\left\{\frac{P^{(2)}(t)}{P(t)}\right\}^n\right]}\sum_{n=0}^{2k-1}\frac{(-1)^n}{\lambda^{n+1}}P^{(n)}(t)\cdot e^{-\lambda\cdot t} \tag{15}$$

Rewriting the summation of Eq. (14) and using the conditions imposed by Eqs. (11) and (12), one has

$$\sum_{n=0}^{2k-1}\frac{(-1)^n}{\lambda^{n+1}}P^{(n)}(t) = \left[\frac{P(t)}{\lambda} - \frac{P^{(1)}(t)}{\lambda^2}\right]\cdot\frac{1-\left\{\frac{P^{(2)}(t)}{\lambda^2\cdot P(t)}\right\}^n}{1-\frac{P^{(2)}(t)}{\lambda^2\cdot P(t)}}. \tag{16}$$

Replacing Eq. (16) in Eqs. (14) and (15), the following expressions for S_1 and S_2 are obtained:

$$S_1 = \frac{\lambda P(t) - P^{(1)}(t)}{\lambda^2 P(t) - P^{(2)}(t)}\cdot P(t). \tag{17}$$

$$S_2 = -\frac{\lambda P(0) - P^{(1)}(0)}{\lambda^2 P(0) - P^{(2)}(0)}\cdot P(0)\cdot e^{-\lambda\cdot t}. \tag{18}$$

In fact, the result is the same for k odd or even (Diaz et al., 2007).

Finally, substituting Eqs. (13), (17), and (18) in Eq. (6) the inverse point kinetics equation for subcritical systems used in this paper is obtained:

$$\rho(t) = \beta + \Gamma + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{\beta P_0}{P(t)}e^{-\lambda t} - \frac{\lambda\cdot\beta}{P(t)} \cdot\left\{\frac{\lambda\cdot P(t) - P^{(1)}(t)}{\lambda^2 P(t) - P^{(2)}(t)}\cdot P(t) - \frac{\lambda\cdot P(0) - P^{(1)}(0)}{\lambda^2 P(0) - P^{(2)}(0)}\cdot P(0)\cdot e^{-\lambda\cdot t}\right\} - \frac{\Lambda\cdot Q}{P(t)}. \tag{19}$$

Eq. (19) allows the reactivity calculation to be made faster when compared with the time needed to numerically solve the integral of the power history, Eq. (7). Apart from that, Eq. (19) does not need the storing of the entire reactor power history, and the calculation can be restarted at any time and the extension is straightforward for six delayed neutron precursors groups (Diaz et al., 2007).