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Estimation of sub-criticality using extended Kalman filtering technique

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ABSTRACT

Reactivity is widely used as the paramount means for defining nuclear reactor status. The measurement of reactivity can be only made in an indirect way. Traditionally, reactivity is estimated by the Inverse Point Kinetics (IPK) method. However, this technique suffers from some serious drawbacks like high sensitivity to reactor parameters and less immunity to noise content in the input signals, hence effective only during power range operation. In this paper, the extended Kalman filter (EKF) technique, which is based on stochastic model of reactor kinetics is proposed for subcriticality estimation in nuclear reactor. The proposed technique can work in noisy environment and modeling errors and uncertainties in parameters do not affect the estimation severely as the feedback gain is continuously adjusted during the estimation process. The performance of proposed technique for the reactivity estimation has been evaluated using power variation data sets collected from a PHWR (Pressurized Heavy Water Reactor) and a research reactor. It has been found that with the application of EKF technique, reactivity in a highly subcritical core can be estimated with reasonable accuracy. The EKF based approach has been found to yield higher accuracy, noise suppression and robustness than done by IPK based approach.

1. Introduction

Reactivity is a much more definitive indication of the status of the core than other parameters such as the reactor power and log rate do (Suzuki and Tsunoda, 1964). On-line measurement and indication of reactivity in a nuclear reactor are very much important from the points of view of monitoring shutdown margin, calibration of safety and control devices, detection of any inadvertent introduction of reactivity into the core, quantification of the worth of fuel bundles, planning of suitable actions during the reactor operation, etc. The reactivity meter facilitates continuous surveillance of the core reactivity status from shutdown through the start-up and power range operation of the reactor.

Many modern reactors are equipped with on-line reactivity meters, which mainly employ techniques like Inverse Point Kinetics (IPK) (Ansari, 1991; Khalafi et al., 2011). However, this technique suffers from some serious drawbacks like high sensitivity to reactor parameters and less immunity to noise content in the input signals hence effective only during power range operation. In IPK technique, the system variables are taken as being deterministic. In reality, however, the neutron density has certain noises that are induced through signal detection, transmission, and amplification, or due to the stochastic nature of the fission process itself. The use of an IPK based reactivity meter for realtime indication of subcriticality is a more challenging problem, essentially due to the difficulties in modeling the neutron source strength with sufficient accuracy, and also due to the relatively large fluctuations in neutron signals measured in subcritical systems (Shimazu et al., 2003; Naing et al., 2005; Shimazu and Naing, 2005). Criticality accidents in recent past have shown impressively that certain subcriticality monitoring devices are essential in nuclear facilities, especially in spent fuel storage bays (Shimazu et al., 2006). The IPK algorithm does not explicitly address the stochastic behavior of the reactor kinetics, and does not have any effective provision for noise elimination, hence demands explicit filtering of the neutronic signals as well as the estimated reactivity. The usual method of smoothening the neutron density is to apply the stochastic neutron density to a low pass filter before using the inverse kinetic relation. The results however are not very accurate because the low pass filter may cut out some of the frequencies passed by the reactor system which is itself a low pass filter, which depend upon power level in the reactor and the filter's time constant.

This is where the modern optimal estimation techniques based on state variable concept, with their inherent ability to work with stochastic process and noisy input signals, serve as promising candidates. The Kalman filter, under Gaussian assumption, is the optimal state estimator for linear dynamic systems. Several studies have been reported on the application of the Kalman filtering technique in the domain of reactivity estimation. Venerus and Bullock (1970) have applied the digital Kalman filtering technique to the estimation of dynamic reactivity, where reactivity was assumed

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to be in the form of polynomial time function with unknown coefficients. However, their study was limited to a linearized reactor model. Racz (1992) proposed a method to estimate small reactivity change as well as neutron density via a bank of Kalman filters in critical reactors. Since the point kinetic model of reactor is nonlinear, a nonlinear filtering method must be applied to it. Shinohara and Oguma (1973) derived a nonlinear filter from the Kalman filter designed for the linearized system by adding a nonlinear feedback to it.

In contrast with earlier approaches, in this paper, the extended Kalman filtering (EKF) technique has been proposed for subcriticality estimation in nuclear reactor. The EKF is an extension of Kalman's original filter into the domain of nonlinear stochastic differential equations. The EKF uses the standard Kalman filter equations to the first-order approximation of the nonlinear model about the last estimate. This is a simple and practical nonlinear filtering algorithm which is applicable to on-line and real-time estimation of reactivity using a small digital computer. This technique is advantageous over the IPK because it can work in noisy environment and modeling errors and uncertainties in parameters do not affect the estimation severely as the feedback gain is continuously adjusted during the estimation process. The feasibility of the EKF method has been verified for subcriticality measurement using actual data collected from a Pressurized Heavy Water Reactor (PHWR) and from a research reactor. The paper is organized as follows. Section 2 explains the techniques for reactivity estimation. A brief overview of reactivity control in PHWR and research reactor is presented in Section 3. The usefulness of EKF technique for reactivity estimation is ascertained in Section 4 by analyzing the data collected. Finally the conclusions are drawn in Section 5.

2. Methods of reactivity estimation

The measurement of reactivity can be made in an indirect way only. It must be deduced from the observation of neutron flux density/power which is caused by reactivity. The reactivity variations taking place in a nuclear reactor are generally due to movement of control rods, changes in core composition as a result of transmutation of fission products, and changes in temperature of fuel, moderator and coolant. It is generally very difficult to develop simple mathematical models for all these phenomena. However, the net reactivity arising due to all such variations is generally required. It is then possible to devise suitable methods for estimation of reactivity in the nuclear reactor.

In the following, IPK and EKF techniques of measuring reactivity are discussed. Both the techniques rely upon the point kinetics model of a nuclear reactor, expressed as

$$\frac{dn}{dt} = \frac{\rho - \beta}{\ell} n + \sum_{i=1}^{m} \lambda_i C_i + S \tag{1}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell} n - \lambda_i C_i, \quad i = 1, 2, \ldots, m, \quad \beta = \sum_{i=1}^{m} \beta_i \tag{2}$$

where ρ denotes the reactivity, n denotes the neutronic power, C_i the concentration of the ith group of delayed neutron precursor and m is the total number of delayed neutron precursor groups, β_i and λ_i are delayed neutron parameters, ℓ denotes the prompt neutron life time and S denotes a neutron source. In the present study six delayed neutron precursor groups are considered, hence $m = 6$. In order to use this model for subcriticality estimation, it is necessary to know the effective strength of the neutron source S. The effective strength of neutron source has been assumed as constant and expressed in term of the initial stable subcriticality as (Shimazu and Naing, 2005).

$$S = \frac{-\rho_0 n_0}{\ell} \tag{3}$$

where n_0 denotes the neutronic power at any subcritical steady state and ρ_0 denotes the corresponding subcriticality obtained from physics computations.

2.1. Estimation based on Inverse Point Kinetics (IPK)

From the point kinetics model (1), (2), the following equation can be derived:

$$\rho = \frac{\ell}{n} \left[\frac{dn}{dt} + \sum_{i=1}^{m} \frac{dC_i}{dt} - S \right]. \tag{4}$$

In general it is convenient to treat discrete dynamic models instead of continuous ones because quantities that come from real observations are discrete. Suppose the samples of neutronic power (measurement) are available as n_k at time instant kT_s, $k = 0, 1, 2, \ldots$, where T_s is sampling interval. Then from (2)

$$C_{i,k} = e^{-\lambda_i T_s} C_{i,k-1} + \frac{1}{\lambda_i} (1 - e^{-\lambda_i T_s}) \frac{\beta_i}{\ell} n_k \tag{5}$$

Further in (4) the derivative can be approximated as

$$\frac{dn}{dt}\Big|_k = \frac{n_k - n_{k-1}}{T_s} \tag{6}$$

$$\frac{dC_{i,k}}{dt}\Big|_k = \frac{C_{i,k} - C_{i,k-1}}{T_s} \tag{7}$$

and thus

$$\rho_k = \frac{\ell}{n_k} \left[\frac{n_k - n_{k-1}}{T_s} + \sum_{i=1}^{m} \frac{C_{i,k} - C_{i,k-1}}{T_s} - S \right] \tag{8}$$

which alongwith (5) is suitable for implementation on a digital computer and it will give estimate of reactivity at different time instants.

When fluctuations in neutron flux are large, such as in highly subcritical regime, the reactivity estimates obtained by such a method show significant fluctuations, since the inverse kinetics equation involves differentiation of neutron density signal with respect to time. This method does not have any effective provision for noise elimination. Hence it is difficult to calculate reactivity using IPK in subcritical operating regime where neutron flux level is quite low and fluctuation of the neutron signal is relatively large.

2.2. Estimation based on the extended Kalman filtering (EKF) technique

The Kalman filter is a predictor–corrector type optimal state estimator for linear dynamic systems with Gaussian noise (Sorenson, 1985). The equations for the Kalman filter fall into two groups: time update equations (predictor equations) and measurement update equations (corrector equations). The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step. The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate.

Although Kalman filter was originally devised for linear systems, nonlinear systems can also be addressed by the Kalman filter through some modifications to it as approximations to the optimal state estimator. These modifications include the extended Kalman filter (EKF) (Cox, 1964), the unscented Kalman filter (UKF) (Julier and Uhlmann, 2004), and the particle filter (PF) (Arulampalam