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Statistical distributions of Transition Fatigue Strength and Transition Fatigue Life in duplex S–N fatigue curves

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ABSTRACT

In recent years, Very-High-Cycle Fatigue (VHCF) behavior of metallic materials has become a major point of interest for researchers and industries. The needs of specific industrial fields (aerospace, mechanical and energy industry) for structural components with increasingly large fatigue lives, up to 10^{10} cycles (gigacycle fatigue), requested for a more detailed investigation on the experimental properties of materials in the VHCF regime.

Gigacycle fatigue tests are commonly performed using resonance fatigue testing machines with a loading frequency of 20 kHz (ultrasonic tests). Experimental results showed that failure is due to cracks which nucleate at the specimen surface if the stress amplitude is above the conventional fatigue limit (surface nucleation) and that failure is generally due to cracks which nucleate from inclusions or internal defects (internal nucleation) when specimens are subjected to stress amplitudes below the conventional fatigue limit. Following the experimental evidence, the Authors recently proposed a new probabilistic model for the complete description of S–N curves both in the High-Cycle Fatigue (HCF) and in the VHCF fatigue regions (duplex S–N curves). The model differentiates between the two failure modes (surface and internal nucleation), according to the estimated distribution of the random transition stress (corresponding to the conventional fatigue limit). No assumption is made about the statistical distribution of the number of cycles at which the transition between surface and internal nucleation occurs (i.e., the Transition Fatigue Life TFL).

In the present paper, the TFL distribution is obtained. The resulting distribution depends on the distance between the HCF and the VHCF regions and on the distribution of the random transition stress. It is also shown that the statistical distribution of the fatigue strength at the median TFL (i.e., the Transition Fatigue Strength TFS) has median which corresponds to the mean transition stress. Finally, a procedure for computing Likelihood Ratio Confidence Intervals (LRCIs) for both the median TFL and the median TFS is given in the paper.

The estimated TFL and TFS distributions can be effectively used for properly choosing the duration of HCF tests in terms of number of cycles and the stress amplitude below which VHCF failures more probably occur. LRCIs for the median TFL and TFS can be usefully computed for assessing uncertainty in the estimation procedure when a limited number of experimental data is available.

A numerical example based on an experimental dataset taken from the literature is provided.

1. Introduction

In recent years, Very-High-Cycle Fatigue (VHCF) test results showed that specimens may also fail at stress amplitudes below the conventional fatigue limit and, therefore, drastically affected the way of modelling fatigue data and designing machine components under VHCF loading conditions [1].

Two distinct failure mechanisms are generally visible in VHCF data plots and, at a stress value near the conventional fatigue limit, plots show a plateau separating the two failure modes. For this reason, the conventional fatigue limit can be considered as a transition stress that differentiates between the two failure modes [2]. In particular, the plateau separating different failure mechanisms represents a transition stress, while the plateau separating finite lives from infinite lives can be considered as a real fatigue limit, if it

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Nomenclature

cdf cumulative distribution function
HCF High-Cycle Fatigue
LRCI Likelihood Ratio Confidence Interval
ML Maximum Likelihood
pdf probability density function
PL Profile Likelihood
rv random variable
TFL Transition Fatigue Life
TFS Transition Fatigue Strength
VHCF Very-High-Cycle Fatigue
$a_{Y|int}, b_{Y|int}, a_{Y|surf}, b_{Y|surf}$ parameters involved in the Basquin's laws
$F_{X|Y=y}, F_{X_l}, F_{X_t}, F_{X_{t_0}}, F_{X_{t_m}}, F_{Y|int}, F_{Y|surf}, F_{Y|X=x}$ cdfs
$f_{X|Y=y}, f_{Y|X=x}$ pdfs
int internal-nucleated failure
$L[\cdot]$ Likelihood function
$PL[\cdot]$ Profile Likelihood function
surf surface-nucleated failure
$TFL_{0.5}$ median TFL
$TFS_{0.5}$ median TFS
x logarithm of the stress amplitude (value)
$x_{t,\alpha}$ α-th quantile of the transition stress
$X|Y=y$ logarithm of the random fatigue strength at a given fatigue life
X_l random fatigue limit
X_t random transition stress
y fatigue life (value)
$y_{t,\alpha}$ α-th quantile of the TFL
$Y|X=x$ random fatigue life at a given stress amplitude
$Y|int$ random fatigue life given that failure is internally-nucleated
$Y|surf$ random fatigue life given that failure is superficially-nucleated
$z_{TFS_{0.5},int}, z_{TFS_{0.5},surf}$ quantiles used for computing the LRCI of $TFS_{0.5}$
α probability value
$\chi^2_{(1;1-\alpha)}$ $(1-\alpha)$-th quantile of the Chi-square distribution with 1 degree of freedom
$\Phi[\cdot], \Phi_{X|int}, \Phi_{X|surf}, \Phi_{X_l}, \Phi_{X_t}, \Phi_{Y|int}, \Phi_{Y|surf}$ standardized Normal cdfs
$\varphi[\cdot], \varphi_{X|int}, \varphi_{X|surf}, \varphi_{X_l}, \varphi_{X_t}, \varphi_{Y|int}, \varphi_{Y|surf}$ standardized Normal pdfs
$\boldsymbol{\theta} = (\theta_1, \boldsymbol{\theta_2})$ parameter set
θ_1 parameter of interest for the PL function
μ_{X_l}, μ_{X_t} mean values
$\sigma_{X_l}, \sigma_{X_t}, \sigma_{Y|int}, \sigma_{Y|surf}$ standard deviations
$\cdot|\cdot$ conditional event
$|\cdot|$ absolute value
$\tilde{}$ parameter estimate

exists [3,4]. Following the experimental evidence, new fatigue life models [2,5–7] were proposed in the literature for the description of S–N curves characterized by two failure modes.

A novel general statistical model, which can take into consideration the two failure modes (duplex S–N curve) and the possible presence of a fatigue limit is described in [8]. The model differentiates between the two failure modes (surface and internal nucleation), according to the estimated distribution of the random transition stress (corresponding to the conventional fatigue limit). No assumption is made about the statistical distribution of the number of cycles at which the transition between surface and internal nucleation occurs (i.e., the Transition Fatigue Life TFL).

In the present paper, the TFL distribution is obtained, according to the statistical model proposed in [8]. The statistical distribution of the fatigue strength at the median TFL (i.e., the Transition Fatigue Strength TFS) is also estimated. Finally, a procedure for computing Likelihood Ratio Confidence Intervals (LRCIs) for both the median TFL and the median TFS is given.

A numerical example, based on an experimental dataset taken from the literature, is provided. The paper shows results obtained in case of a duplex S–N curve with fatigue limit.

2. Methods

In [8], a unified statistical model for various types of S–N curve was defined. In Section 2.1, the particular case of duplex S–N curves is recalled. The model is able to take into account the possible presence of a fatigue limit. According to the approach proposed in [9] and commonly adopted in the literature (see e.g., [10–12]), the fatigue strength distribution for a given number of cycles is also derived in Section 2.1.

Given the fatigue life and the fatigue strength distributions, the procedure for the estimation of the TFL and TFS distributions is presented in Section 2.2. Finally, Section 2.3 shows the steps for computing the LRCIs for both the median TFL and the median TFS.

2.1. Duplex S–N curve: statistical distributions of fatigue life and fatigue strength

In case of duplex S–N curve with fatigue limit, the cumulative distribution function (cdf) of the fatigue life Y (i.e., logarithm of the number of cycles to failure) for a given logarithm of the stress amplitude x can be expressed as [8]:

$$F_{Y|X=x} = F_{Y|surf}F_{X_t} + F_{Y|int}F_{X_l}(1 - F_{X_t}), \tag{1}$$

where $F_{Y|surf}$ is the cdf of the fatigue life if crack nucleates superficially (i.e., of the random variable (rv) $Y|surf$), $F_{Y|int}$ is the cdf of the fatigue life if crack nucleates internally (i.e., of the rv $Y|int$), F_{X_t} is the cdf of the logarithm of the transition stress (i.e., of the rv X_t) and F_{X_l} is the cdf of the logarithm of the fatigue limit (i.e., of the rv X_l).

$F_{Y|X=x}$ given in Eq. (1) depends on the cdfs of the continuous rvs $X_l, X_t, Y|int$ and $Y|surf$. According to the literature [13–16] on the fatigue strength, both X_l and X_t can be assumed as Normal distributed (i.e., the fatigue limit and the transition stress are Log-Normal distributed). In particular, let X_l have mean value μ_{X_l} and standard deviation σ_{X_l}, and X_t have mean value μ_{X_t} and standard deviation σ_{X_t}, then:

$$F_{X_l} = \Phi\left[\frac{x - \mu_{X_l}}{\sigma_{X_l}}\right], \tag{2}$$

and

$$F_{X_t} = \Phi\left[\frac{x - \mu_{X_t}}{\sigma_{X_t}}\right], \tag{3}$$

where $\Phi[\cdot]$ is the standardized Normal cdf.

In the literature [13–15], different types of continuous distribution have been proposed for the number of cycles to failure. Usually, either a 2-parameter Weibull distribution or a Log-Normal distribution are used for the cycles to failure. Without loss