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Evolution of crack paths and compliance in round bars under cyclic tension and bending

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ABSTRACT

The aim of this paper is to calculate how the surface crack front and the dimensionless compliance evolve in cylindrical bars containing a transverse edge crack subjected to cyclic tension or bending with different initial crack geometries (crack depths and aspect ratios). A Java computer program was developed and implemented to obtain the crack path evolution (changes of crack depth and aspect ratio), as well as the progress of mechanical compliance, by discretizing the crack front (characterized with elliptical shape) and calculating the crack front changes on the basis of the Paris law, and using a three-parameter stress intensity factor (SIF). The results show that in fatigue crack propagation, relative crack depth influences more on dimensionless compliance than the aspect ratio, because the crack front tends to converge when the crack propagates from different initial geometries, showing greater values of dimensionless compliance for tension than for bending. Furthermore, during fatigue crack growth, materials with higher values of the exponent of the Paris law produce slightly greater dimensionless compliance and a better convergence between the results for straight-fronted and circular initial cracks.

1. Introduction

Cylindrical structural elements are widely used in mechanical and civil engineering in the form of wires, ropes, chords, strands, cables, shafts, etc. These elements can be subjected to cyclic loading and harsh environments, so that the initiation and propagation of cracks by mechanical fatigue or corrosion-fatigue could be a very serious problem from the point of view of fracture mechanics, damage tolerance and structural integrity during their service life.

The phenomenon of propagation of cracks contained in the transverse section of cylindrical round bars has been studied, from the experimental and numerical viewpoints, following several criteria [1–7]. Among them, the iso-*K* criterion, cracks intersecting the free lateral surface at 90° angles, propagation on the basis of the Paris law [8] or the modified Forman model [9]. Most of these criteria predict a preferential path during crack growth. In notched bars, the crack shape change is significant at the early stage of crack growth, rapidly leading to a sickle-shaped crack front [3,10] due to the stress concentration caused by the notch.

The stress intensity factor SIF *K* in cracked round bars (with elliptical crack front) under tension or bending has been calculated in the past by means of different numerical methods for the stress–strain analysis (mainly finite element calculations and boundary integral equation approaches) and a wide collection of procedures to obtain the SIF: stiffness derivative method, virtual crack extension techniques, opening normal stress fitting, extrapolation method…

In the framework of the computational approaches described in the previous paragraph, many *K*-solutions for edge cracks in round bars appear in the scientific literature [11–13]. The most sophisticated are those including three parameters (the crack depth, the crack aspect ratio and the position of the specific point at the crack front) [13], they being very adequate for analysis of crack shape evolution in fatigue crack propagation [14].

With regard to the mechanical compliance, in cracked round bars under tension or bending, it depends on the loading mode, the crack depth and the crack aspect ratio [4,15]. In addition, the compliance also depends on the boundary conditions, i.e., if the bar is loaded under free or constrained ends and, when the crack grows by fatigue, on the exponent of the Paris law of the material [16].

This paper describes a numerical procedure to determine the crack front evolution in semi-elliptical surface cracks in cylindrical round bars (based on the Paris–Erdogan law) and to analyze how the mechanical compliance evolves under cyclic fatigue loading. Different materials were taken into account (represented by diverse Paris coefficients *m* of 2, 3 and 4), distinct initial crack geometries were considered (circular and quasi-straight), a set of

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three initial crack depths (one tenth, one third and half of the diameter) and two loading modes (tension and bending). The article goes further in the analyses provided in previous papers [7,16,17] by taking into account other loading cases (bending moment) and considering also the compliance evolution in the computations. In addition, comparison between tension and bending is presented now.

2. Numerical modeling

A Java computer program was designed to predict how the crack front evolves according to the Paris law, and how the mechanical compliance changes under tensile and bending fatigue (Fig. 1).

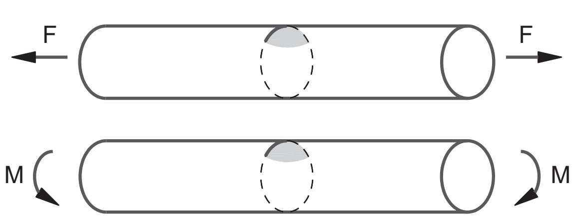

Fig. 1. Cracked bar under tension loading or bending moment.

2.1. Crack shape evolution

The modeling procedure describing in the present paper is based on the assumption that the transverse crack front may be represented by means of an elliptical shape centered at the periphery of the cylinder (Fig. 2), as described elsewhere [17]. The method consists of the following steps:

2.1.1. Division of the semi-elliptical crack front in parts characterized by equal length

The crack front was discretized by dividing it in set segments with exactly the same length using the Simpson's rule [18]. The following change of coordinates is needed:

$$\begin{cases} x = b\cos\theta \\ y = a\sin\theta \end{cases} \tag{1}$$

The relationship between the Cartesian coordinates (x, y) and the angular coordinate θ is given in Fig. 3.

The length L of the elliptical arc corresponding to each crack front is obtained through the following equations:

$$L = \int_{\theta_b}^{\theta_e} \sqrt{x^2+y^2}\,d\theta = \int_{\theta_b}^{\theta_e} \sqrt{b^2\sin^2\theta + a^2\cos^2\theta}\,d\theta \tag{2}$$

where θ_b and θ_e are the initial (beginning) and final (end) angles (angular coordinates for calculations, cf. Fig. 3) of the arc of ellipse whose length is calculated. The Simpson's rule [18] was used to obtain the integral of Eq. (2) by dividing the interval [θ_b,θ_e] in 4000 parts.

Now the total length L of the semi-elliptical crack front is divided in z equal parts. To obtain the θ-coordinate defining the end of each new part of the ellipse θ_i, the following functions are defined,

$$F(\theta) = \int_{\theta_b}^{\theta} \sqrt{b^2\sin^2\theta + a^2\cos^2\theta}\,d\theta \tag{3}$$

$$f(\theta) = \sqrt{b^2\sin^2\theta + a^2\cos^2\theta} \tag{4}$$

so as to determine approximately $F(\theta)$ by using the first term of the Taylor series expansion, as follows,

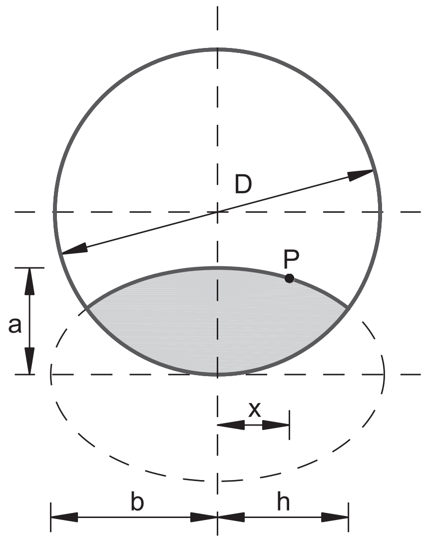

Fig. 2. Elliptical crack model used by Shin and Cai [13].

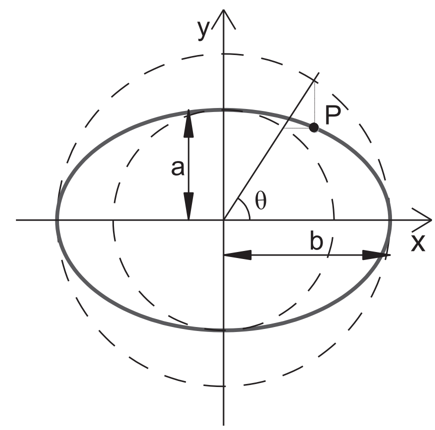

Fig. 3. Relationship between Cartesian coordinates (x,y) and angular coordinate (θ).

$$F(\theta) \approx (\theta - \theta_b) f(\theta_b) \tag{5}$$

which allows one to find an initial value for θ_i of a given arc i from the initial one θ_b, by using the following expression for $F(\theta_i)$,

$$F(\theta_i) = L\frac{i}{z} \tag{6}$$

and each final value θ_i can be calculated (Newton–Raphson method [18]), in an iterative manner, as follows,

$$\theta_i^j \approx \theta_i^{j-1} + \frac{L\frac{i}{z} - F\left(\theta_i^{j-1}\right)}{f\left(\theta_i^{j-1}\right)} \tag{7}$$

Fig. 4 shows the functions $F(\theta)$ and $f(\theta)$ for a semiellipse, where $F(\theta)$ is a non-linear function.

2.1.2. Advance of specific points at the crack front

After that, every single point was shifted, perpendicularly to the previous crack front, and following the Paris–Erdogan law [8] to represent the cyclic crack growth under fatigue,

$$\frac{da}{dN} = C\Delta K^m \tag{8}$$

The maximum crack depth increment $\Delta a(\max) \equiv \max\ \Delta a_i$ was maintained constant during the process of crack advance. The shifting of each point at the crack front, Δa_i, was calculated as follows,