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# A discrete stress–strength interference model based on universal generating function

Zong-Wen An[a,b], Hong-Zhong Huang[a,*], Yu Liu[a]

[a]School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 610054, PR China
[b]School of Mechatronics Engineering, Lanzhou University of Technology, Lanzhou, Gansu 730050, PR China

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**Abstract**

Continuous stress–strength interference (SSI) model regards stress and strength as continuous random variables with known probability density function. This, to some extent, results in a limitation of its application. In this paper, stress and strength are treated as discrete random variables, and a discrete SSI model is presented by using the universal generating function (UGF) method. Finally, case studies demonstrate the validity of the discrete model in a variety of circumstances, in which stress and strength can be represented by continuous random variables, discrete random variables, or two groups of experimental data.


*Keywords:* Discrete model; Stress–strength interference; Universal generating function; Component reliability

## 1. Introduction

The stress–strength interference (SSI) model has been widely used for reliability design of mechanical component. In this model, if stress on a component and strength of a component are denoted by $S_1$ and $S_2$, respectively, the component reliability denoted by $R$ is then defined as

$$R = Pr(S_2 > S_1). \tag{1}$$

Eq. (1) is the most basic expression of the SSI model, which means that the component reliability is taken as the probability that the strength is larger than the stress. Furthermore, if both stress and strength are treated as continuous random variables (r.v.) and their probability density functions (p.d.f.), denoted by $f_1(S_1)$ and $f_2(S_2)$ respectively, Eq. (1) can be rewritten as the following formulas:

$$R = \int_{-\infty}^{\infty} f_1(S_1)\left[\int_{S_1}^{\infty} f_2(S_2)\,\mathrm{d}S_2\right]\mathrm{d}S_1 \tag{2a}$$

or

$$R = \int_{-\infty}^{\infty} f_2(S_2)\left[\int_{-\infty}^{S_2} f_1(S_1)\,\mathrm{d}S_1\right]\mathrm{d}S_2. \tag{2b}$$

For the sake of clarity, Eq. (2) can be called the continuous SSI model.

Theoretically, we can calculate the reliability or unreliability of a component analytically or numerically on the basis of Eq. (2) when the p.d.f. of stress and strength are available. However, in engineering practice, it is often difficult to know the exact distribution of stress and strength. In most cases, what we can obtain is the finite experimental data regarding stress and strength only. Consequently, it is necessary to study the approximate methods when calculating component reliability, and in this regard, some efforts have been made by many researchers.

Kapur [1] devised an approach for determining the bounds on exact unreliability, and this approach required only information regarding the subinterval probabilities

*Abbreviations (the singular and plural of an abbreviation is always spelled the same):* r.v., random variable; p.d.f., probability density function; p.m.f., probability mass function; SSI, stress–strength interference; UGF, universal generating function

*Corresponding author. Tel.: +86 28 8320 6916; fax: +86 28 8320 6828.
*E-mail address:* hzhuang@uestc.edu.cn (H.-Z. Huang).

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within an interference region. To improve the accuracy of calculation, Park and Clark [2] modified Kapur's formulation on the quadratic programming problem and presented a solution to this problem. Shen [3] proposed another empirical approach to computing the unreliability bounds based upon the subinterval probabilities of the stress and strength in the interference region. Wang and Liu [4] presented a multiple-line-segment method of implementing the SSI model when only discrete interval probabilities of stress and strength inside an interference region are available. Guo and Li [5] presented an algorithm for computing the unreliability bounds based on an improved Monte Carlo method. Wang and Liu [6] presented an approach to calculate fuzzy unreliability of a component/system. In this approach the p.d.f. of stress and strength were approximated by piecewise fuzzy line-segments that were expressed by linear fuzzy polynomials, and the discrete interval probabilities were treated as fuzzy numbers. Additionally, summarizing the research results from diverse disciplines, Kotz et al. [7] generalized the stress–strength model and provided the computation methods on the basis of maximum likelihood estimation.

In this paper, unlike those in the continuous SSI model, stress and strength are treated as discrete r.v. and their probability mass functions (p.m.f.) are represented by universal generating functions (UGF). According to the basic definition of component reliability expressed by Eq. (1), a discrete SSI model is established. This model can be utilized for calculating component reliability under the following circumstances:

(1) stress and strength are discrete r.v.,
(2) stress and strength are continuous r.v.,
(3) the distributions of stress and strength are unavailable but their frequency distributions are known based on experiment data.

This paper begins with a description of the UGF method that is the modeling tool of discrete SSI system and proceeds with a building model of SSI system by employing UGF method. Finally, two cases are studied to demonstrate the effectiveness and advantage of the discrete SSI model.

## 2. Brief description of UGF method

In this section, emphasis is put on the basic process but not the fundamental mathematics of UGF method. The concept of UGF was first introduced by Ushakov [8]. Then in a series of research work by Lisnianski and Levitin, the UGF method has been applied to reliability analysis and optimization of multi-state system [9,10].

### 2.1. UGF of discrete random variable

Suppose that a discrete r.v. $X$ has a p.m.f. characterized by the vector $\boldsymbol{x}$ consisting of the possible values of $X$ and the vector $\boldsymbol{p}$ consisting of the corresponding probabilities, which can be formulated by the following expressions:

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_k),$$

$$\boldsymbol{p} = (p_1, p_2, \ldots, p_k),$$

$$p_i = Pr(X = x_i), \quad i = 1, 2, \ldots, k.$$

Based on the basic principle of UGF method, the p.m.f. of discrete r.v. $X$ can be represented by a polynomial function of variable $z$, $u_X(z)$, that relates the possible values of $X$ to the corresponding probabilities as

$$u_X(z) = p_1 z^{x_1} + p_2 z^{x_2} + \cdots + p_k z^{x_k} = \sum_{i=1}^{k} p_i z^{x_i}. \tag{3}$$

It should be mentioned that, for an arbitrary discrete r.v., its UGF is uniquely determined by its p.m.f. This means that a one-to-one correspondence exists between the p.m.f. and UGF of a discrete r.v.

### 2.2. UGF of function of discrete random variables

Consider $n$ independent discrete r.v. $X_1, X_2, \ldots, X_n$. Let the UGF of each r.v. be $u_{X_1}(z), u_{X_2}(z), \ldots, u_{X_n}(z)$, respectively, and $f(X_1, X_2, \ldots, X_n)$, an arbitrary function of variables $X_1, X_2, \ldots, X_n$. Then, by employing composition operator $\otimes$, the UGF of function $f(X_1, X_2, \ldots, X_n)$, $u_f(z)$, can be obtained as follows:

$$u_f(z) = \otimes(u_{X_1}(z), \; u_{X_2}(z), \; \ldots, \; u_{X_n}(z)). \tag{4}$$

### 2.3. Definition and properties of composition operator $\otimes$

Without loss of generality, we still consider $n$ independent discrete r.v. $X_1, X_2, \ldots, X_n$ and an arbitrary function $f(X_1, X_2, \ldots, X_n)$. Suppose that the number of possible values of each r.v. are $k_1, k_2, \ldots, k_n$, respectively. According to Eq. (3), the UGF of individual r.v. can be obtained as follows:

$$u_{X_1}(z) = \sum_{j_1=1}^{k_1} p_{1j_1} z^{x_{1j_1}},$$

$$u_{X_2}(z) = \sum_{j_2=1}^{k_2} p_{2j_2} z^{x_{2j_2}},$$

$$\ldots$$

$$u_{X_n}(z) = \sum_{j_n=1}^{k_n} p_{nj_n} z^{x_{nj_n}}.$$

To obtain the UGF of function $f(X_1, X_2, \ldots, X_n)$, composition operator $\otimes$ is defined as

$$\otimes\left(\sum_{j_1=1}^{k_1} p_{1j_1} z^{x_{1j_1}}, \sum_{j_2=1}^{k_2} p_{2j_2} z^{x_{2j_2}}, \ldots, \sum_{j_n=1}^{k_n} p_{nj_n} z^{x_{nj_n}}\right) = \sum_{j_1=1}^{k_1}\sum_{j_2=1}^{k_2}\cdots\sum_{j_n=1}^{k_n}\left(\prod_{i=1}^{n} p_{ij_i} z^{f(x_{1j_1}, x_{2j_2}, \ldots, x_{nj_n})}\right). \tag{5}$$