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An adaptive mesh refinement approach for average current nodal expansion method in 2-D rectangular geometry

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ABSTRACT

The aim of this work is to develop a spatially adaptive coarse mesh strategy that progressively refines the nodes in appropriate regions of domain to solve the neutron balance equation by zeroth order nodal expansion method. A flux gradient based a posteriori estimation scheme has been utilized for checking the approximate solutions for various nodes. The relative surface net leakage of nodes has been considered as an assessment criterion. In this approach, the core module is called in by adaptive mesh generator to determine gradients of node surfaces flux to explore the possibility of node refinements in appropriate regions and directions of the problem. The benefit of the approach is reducing computational efforts relative to the uniform fine mesh modeling. For this purpose, a computer program ANRNE-2D, Adaptive Node Refinement Nodal Expansion, has been developed to solve neutron diffusion equation using average current nodal expansion method for 2D rectangular geometries. Implementing the adaptive algorithm confirms its superiority in enhancing the accuracy of the solution without using fine nodes throughout the domain and increasing the number of unknown solution. Some well-known benchmarks have been investigated and improvements are reported.

1. Introduction

The diffusion approximation has successfully been used for the analysis of the current generation of Light Water Reactors (LWRs) because the transport equation is very costly to solve for reactor core-sized problems. Conventional difference techniques for the solution of multi-group diffusion equation are principal tools used in design calculations for reactor cores. However, there is a demand for faster calculations to help fuel loading schemes and adjustment of excess reactivity in the system to optimize the power distribution. Such operational requirements have led to development of nodal methods as a fast tool for reactor calculations, Ackroyd (1986). These methods were developed in the 1970s for numerical reactor treatment, especially for neutron diffusion equation applications. Nodal methods share some attractive features of both finite difference and finite element methods. From the finite element method they borrow a piecewise continuous polynomial function for flux approximation over an element. With finite difference method, they share the fact that the final set of algebraic equations is usually quit sparse and well structured. Nodal methods deal with average or integral quantities over the node, Putney (1984, 1987). As a consequence, nodal methods involve fewer unknowns than the finite difference method.

There has been significant progress in the use of nodal methods to provide fast diffusion theory calculations in reactor physics calculations. Nodal equations are obtained by the integrating multigroup diffusion equation over a homogeneous region, node, and then relating the net currents across the surfaces to the outgoing and incoming partial currents. Spatial coupling coefficients are then used to relate the average fluxes and the average outgoing partial currents on the surfaces. The spatial coupling coefficients can be defined in terms of the net currents across a surface and the average fluxes in two adjacent nodes.

The nodal expansion method, NEM, is one of the nodal class method in which the average interface partial currents are treated explicitly, Wagner et al. (1977) and Finnemann et al. (1977). A polynomial expansion of flux within the node is used to relate the partial currents on the surfaces of a node to the flux within the node. Weighted residual procedures are used to calculate the coefficient expansions. The method is developed by integration the neutron diffusion equation over a node and evaluating the resulting integral by expanding the flux in products of higher order polynomials. The simplest case is to expand the flux in quadratic polynomial with coefficients given in terms of the fluxes at the center of the node and at interface midpoints. Surface fluxes are determined by inverting a tri-diagonal matrix obtained by imposing flux and net current continuity across the node surfaces in each coordinate direction. Putney (1984) employed a similar technique in which the nodal expansion is fitted to the node and node surface

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average fluxes. This enables the nodal equations to be written in terms of the average node fluxes.

NEM plays an important role in nodal diffusion calculations, Finnemann et al. (1977). Furthermore, the NEM presents very suitable features, such as simplicity, computational efficiency and acceptable accuracy for most realistic nuclear reactor simulations, Pimenta De Abreu et al. (1995). Initially, zeroth and higher order of nodal expansion methods were described and investigated in documents such as Langenbuch et al. (1975, 1977a, 1977b), Wagner et al. (1977) and Turinsky (1994).

Recently, zeroth order nodal expansion methods including average current, average flux, point current and point flux methods have been investigated and reported by Poursalehi et al. (2012) and results of methods have been compared together in terms of accuracy and convergence behavior using fission source weightings. In the zeroth order variants of NEM, results usually experience some errors in the coarse mesh calculations due to high flux gradient existence in some elements such as periphery FAs and reflectors. In order to obtain more accuracy, one can increase the order of solution or/and decrease the mesh sizes in the whole of a domain, Poursalehi et al. (2012).

The specific approximations which contribute to the errors observed in nodal diffusion methods can be identified into four primary effects: a spatial discretization effect, a spatial homogenization effect, a group collapsing effect, and finally a transport effect. The quantification analysis of the four effects under various possible environments is very important to isolate the source of errors and to address direction for improving in core calculations, Downar and Lewis (2005). Spatial discretization effects may be optimized by using an adaptive grids strategy which starts with obtaining an approximate solution for a coarse grid. The quality of this solution is then assessed by estimating the currents in all surfaces of nodes. This estimation is then employed to modify the nodes with high leakage; the element modification is generally carried out using a refinement of the grid size (h-type). Several error estimations can be found in the literature, e.g. Zienkiewics and Zhu (1997) and McRae (2001). One of the common error estimations is based on obtaining a residual error over an element. Bank and Weiser (1985) have used this scheme for linear elliptic problems. Since this scheme depends on the governing equations of the problem, it is often more accurate and robust than the other schemes (Babuska, 1994; Carey, 1997; Abbassi et al., 2011).

In the field of neutronics and reactor applications, some authors have also considered mesh adaptive for P_N and diffusion approximations. Zhang and Lewis (2001, 2002) presented a refinement technique for the inter-element approximation in a primal hybrid finite element technique; diffusion, and P_N, results were given for a 2-D one-group problem. Ragusa (2004, 2008) employed an error indicator based on estimating the second derivatives of the numerical solution and applied the resulting method to multi-group diffusion problems; their error indicator was based on the interpolation error of linear finite elements, Frey and Alauzet (2005) and therefore limiting the application to these elements. More recently, Wang and Ragusa (2009) applied the hp-adaptation concept to the multi-group diffusion equations, where both the local polynomial degree of shape functions (p-type) and the local cell size (h-type) are selected adaptively; nonetheless, error estimators for the combined hp adaptation are less mature and, as a consequence, these authors used the difference between a finer mesh solution $\mathbf{\Phi}_{\text{fine}}$ and a coarser mesh solution $\mathbf{\Phi}_{\text{coarse}}$ to drive the refinement; their results included mesh adaptation for 1-D multi-group and 2-D 1-group diffusion problems using finite element solution, Wang (2006). An approach based on a uniformly distributed fine mesh can be too costly, in both memory and CPU time, to provide a reasonably accurate numerical solution. With a posteriori error estimation, Wang (2009), the zones with larger errors are selected for refinement. Hence, it is possible to control the numerical error in an automated succession of computations performed on locally adapted meshes. This leads to high resolution numerical solutions that can be obtained with fewer unknowns and smaller CPU times than the more pedestrian approach based on uniform mesh refinement. Because the number of unknowns associated with an adapted mesh is orders of magnitude smaller than the number of unknowns for a uniform mesh yielding about the same required accuracy and the total computational time of the whole adaptive mesh refinement, AMR, procedure is typically smaller than the time needed for one calculation on a uniform fine mesh. The memory usage is reduced significantly as well.

In this work, an adaptive coarse node h-refinement technique is implemented for zeroth order average current nodal diffusion method. For large multi-dimensional realistic problems, nodal methods treat diffusion equation with nodes as large as fuel assemblies. In the zeroth order of nodal expansion method, due to large fuel assemblies, solutions may be encountered with significant errors in the power distribution. The errors may be reduced by increasing flux expansion degree or refining nodes. In this paper an adaptive node refinement nodal expansion code (ANRNE-2D) is developed for treatment of multi-group, two dimensional diffusion equations to achieve higher accuracy relative to uniform coarse mesh scheme.

Outline of the remainder of this paper is as follows: in Section 2, summary of zeroth order average current nodal expansion method for 2D rectangular nodes is given, Section 3, the adaptive nodal h-refinement approach is discussed and proposed algorithm is presented and in Section 4, some popular benchmarks are investigated and the numerical results of applying adaptive solution are given.

2. Derivation of nodal equations

In the nodal expansion methods the multi-group neutron diffusion equation is effectively solved by representing the neutron flux in each node by a polynomial expansion, and using a combination of weighted residual equations, i.e.:

$$\int_{\Pi^m} W_{gu}^{[k]} \left\{ -div D_g \nabla \phi_g + \Sigma_{tg}\phi_g - \sum_{g'=1}^{G} \Sigma_{gg'}\phi_{g'} - \frac{\chi_g}{k_{eff}} \sum_{g'=1}^{G} \upsilon \Sigma_{fg'}\phi_{g'} \right\} d\Pi^m = 0 \quad m = 1,2,\ldots,M, \quad g = 1,2,\ldots,G, \quad k = 0,1,2,\ldots,K, \quad u = x,y \tag{1}$$

where m, g, u and k are node number, energy group, cartesian axes and order of weighted diffusion equation respectively along continuity conditions to determine its coefficients.

In the zeroth order variants of the methods, the nodal flux expansions are chosen to be quadratic and $W_{gu}^{[0]} = 1$. In this case, the necessary coefficients can be determined by forcing the expansions to satisfy the zeroth order ($k = 0$) weighted diffusion Eq. (1), i.e. the integral neutron balance in the nodes, and the continuity of neutron flux and net normal neutron current on their surfaces. Four variations basic approach are possible: the point current method, the point flux method, the average current method and the average flux method. The point methods are based on a nodal flux expansion which is fitted to the values of the neutron flux at the center points of the node and its surfaces. The average methods, on the other hand, are based on an expansion which is fitted to the average values of the flux in the node and its surfaces. In the flux methods, the nodal integrated neutron diffusion equation is imposed directly on the nodal flux expansion. In the current methods however, the nodal integrated neutron continuity equation is instead imposed. The surface fluxes are then replaced by surface partial currents and the diffusion approximation is introduced to derive an additional set of equations to calculate the