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Technical Note

Fractional-space law for the neutron current density

G. Espinosa-Paredes [a,*], R. Vázquez-Rodríguez [a], E. del Valle Gallegos [b], G. Alonso [c], N.M. Moghaddam [d]

[a] Área de Ingeniería en Recursos Energéticos, Universidad Autónoma Metropolitana-Iztapalapa, Av. San Rafael Atlixco 186 Col. Vicentina, México, D.F. 09340, Mexico
[b] Dpto. de Física, Instituto Politécnico Nacional, Av. Instituto Politécnico Nacional s/n, Col. Lindavista, México, D.F. 07738, Mexico
[c] Instituto Nacional de Investigaciones Nucleares, Carretera México-Toluca s/n, 52750, Municipio de Ocoyoacac, Estado de México, Mexico
[d] Department of Nuclear Engineering and Physics, Amirkabir University of Technology (Tehran Polytechnic), Hafez street, Tehran, Iran

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ABSTRACT

In this work, a fractional constitutive law (FCL) for the average neutron motion in nuclear reactors is presented. The FCL has a fractional exponent of the differential operator, which is the new unknown. The application of the FCL in the neutron balance equation leads to the fractional diffusion equation. The fractional exponent was evaluated using detrended fluctuation analysis (DFA). The DFA technique is based on the random walk theory and was applied to a power signal of the Forsmark Boiling Water Nuclear Reactor. The results show that the fractional exponent is between 0.1881 and 0.9399 for stationary operation, load changes and unstable conditions.

1. Introduction

In a previous work (Espinosa-Paredes et al., 2008) it was studied the process of neutron diffusion, which takes place in a highly heterogeneous hierarchical configuration as is illustrated in Fig. 1. The scaling considered for these authors are: (1) a nuclear reactor core, (2) fuel assembly which is an array of the fuel cell, and (3) the array of the fuel rod. Normally these arrays (assemblies, cells and rods) are periodic with an anisotropy characterized by the nominal array geometry (Todreas and Kazimi, 1990). In order to describe the neutron transport process in this system, the neutron diffusion concept is a tool commonly used to understand the complex behavior of the neutrons *average* motion. Most reactor studies treat the neutron motion as a diffusion process, where it is assumed that neutrons motion *in the average* tends to diffuse from regions of high neutron flux to regions of low neutron flux, i.e., regions from high linear momentum to regions of lower linear momentum. The treatment of the neutron transport as a diffusion process has only limited validity because neutron tends to stream at relatively large distances between interactions.

The diffusion theory provides a strictly valid mathematical description of the neutron flux when the following assumptions are satisfied: (1) Absorption much less than scattering, (2) smooth spatial variation of the neutron distribution in the scattering media, and (3) isotropic scattering (Stacey, 2004). The first condition is satisfied for most of the moderating and structural materials found in a nuclear reactor, but not for the fuel and control elements. The second condition is satisfied a few mean free paths away from the boundary of large homogeneous medium with relatively uniform source distribution. The third condition is satisfied for scattering from heavy atomic mass nuclei.

With the idea of improving the diffusion theory, a nuclear reactor diffusion equation for the large scale was obtained by Vázquez-Rodríguez et al. (2009) using the volume averaged method (VAM), and it was obtained the *extended linear neutron diffusion equation* (LENDE). The VAM was used as a homogenization method whose starting point is the classical neutron diffusion model. The VAM is comparable to the cell-averaging technique. This is a traditional method of homogenization used in nuclear engineering, which includes the disadvantage factor, ξ, used to obtain the effective cross sections at a cell level, i.e., ξ is a weighting parameter in a spatial cell (Duderstadt and Hamilton, 1976). The consequences of the formulation of the large scale were analyzed by Espinosa-Paredes and Vázquez-Rodríguez (2011). These authors, considered a planar source located in the origin of a semi-infinite homogeneous medium to study the effects of the correction terms of the LENDE. The results obtained with the LENDE approximation were compared with an analytical benchmark based on one-dimensional transport theory. The comparison of the results demonstrate the excellent tendency and agreement, between the linear-extended diffusion theory and the reference transport theory calculation, which implies that the correction terms of the LENDE, are physically acceptable.

* Corresponding author.
E-mail address: gepe@xanum.uam.mx (G. Espinosa-Paredes).

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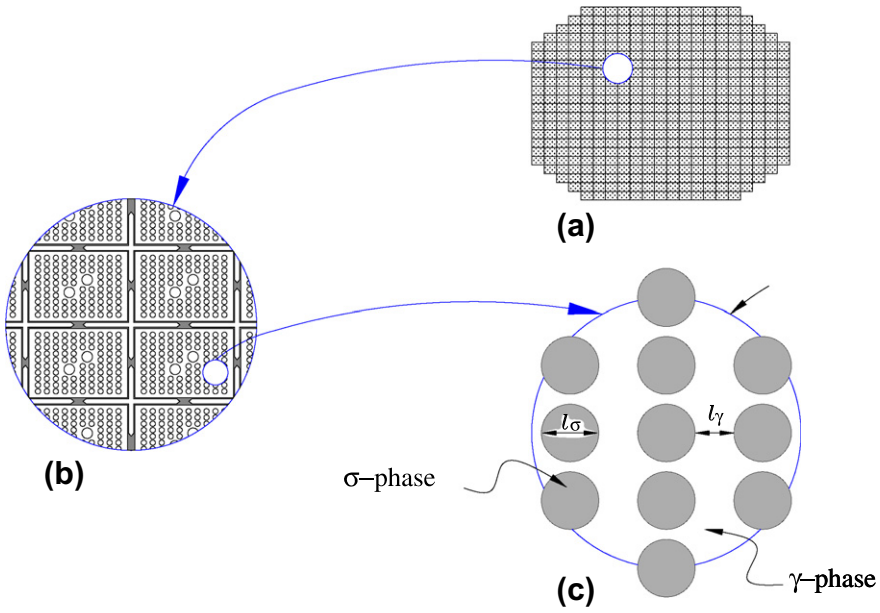

Fig. 1. Characteristic lengths of the system. (a) Nuclear reactor core; (b) Scale I: fuel assembly; (c) Scale II: array of the fuel rod.

Other approximation to improve some of the limitations of the theory of diffusion was proposed by Espinosa-Paredes et al. (2008). These authors consider the following a fractional constitutive law for the neutron current (**J**):

$$\tau^\kappa \frac{\partial^\kappa \mathbf{J}_\gamma}{\partial t^\kappa} + \mathbf{J}_\gamma = -D_\gamma \nabla \phi_\gamma, \qquad \text{for } 0 < \kappa < 1 \tag{0}$$

where the fractional derivative operator $\partial^\kappa/\partial t^\kappa$ is defined in the Riemann–Liouville's sense (Oldham and Spanier, 1974) and κ is the anomalous diffusion exponent. In the limit when $\tau \to 0$, the Fick's law is recovered, (i.e., $\mathbf{J}_\gamma = -D_\gamma \nabla \varphi_\gamma$) and when $\kappa \to 1$ a finite wave propagation velocity for the diffusion process is obtained (as is defined by Chen et al., 2008), and $\tau = \frac{3D_\gamma}{\upsilon}$. It is important to consider that in time-dependent problems, the diffusion equation fails to describe the front of streaming generating from a source, even for a highly scattering medium due to the parabolic nature of the diffusion equation (infinite particle velocity). Then, the P_1 approximation for the Boltzmann equation, which gives rise to the Telegrapher's equation, replaces this feature of an infinite velocity with a wrong finite velocity (given by $\frac{\upsilon}{\sqrt{3}}$), in this sense the proposal of the constitutive law given by Eq. (0) becomes relevant.

The consequences of the Eq. (0) were studied by Espinosa-Paredes and Polo-Labarrios (2012). These authors presented a new approximation from the solution of the time-dependent Boltzmann equation, which includes a fractional constitutive equation of the neutron current density (Eq. (0)), for a general medium. The propagation velocity found by these authors with the fractional constitutive law is $\frac{\upsilon}{\sqrt{3^\kappa}}$. The results of this approximation were compared with the exact solution and Hailer's approximation (Heizler, 2010).

In this work we explore a fractional constitutive law of the neutron current density that considers effects that modify the neutron diffusion theory. The fractional diffusion model developed in this work can be applied where large variations on neutron cross sections normally preclude the use of the classical neutron diffusion equation, specifically, the presence of strong neutron absorbers in the fuel, and control rods to force shutdowns of the reactor.

2. Preliminaries

Consider the processes of collision and reaction in a reactor core with the characteristic length scales given in Fig. 1, where the material fuel is dispersed in lumps within the moderator. It is stressed that this figure represents only one of a variety of spacer designs now in use. We assume that in the highly heterogeneous configuration there are only "two materials" present in the system; namely, the fuel (σ) and moderator (γ). In this paper, in order to illustrate the analysis of neutron diffusion and non-diffusion processes we assume that all the neutrons in the reactor have the same speed and that the angular flux is only linearly anisotropic. Then, the conservation equation that governs the neutron collision and reaction processes in the moderator (γ) in this system, as well as the initial conditions and boundaries at interfaces are given by (Duderstadt and Hamilton, 1976):

γ-moderator

$$\frac{1}{\upsilon}\frac{\partial \phi_\gamma}{\partial t} + \nabla \cdot \mathbf{J}_\gamma(\mathbf{r},t) + \Sigma_{a\gamma}(\mathbf{r})\phi_\gamma(\mathbf{r},t) = S_\gamma(\mathbf{r},t) \tag{1}$$

$$\frac{1}{\upsilon}\frac{\partial \mathbf{J}_\gamma(\mathbf{r},t)}{\partial t} + \frac{1}{3}\nabla \phi_\gamma(\mathbf{r},t) + \Sigma_{tr\gamma}(\mathbf{r})\mathbf{J}_\gamma(\mathbf{r},t) = \mathbf{S}_{\gamma 1}(\mathbf{r},t), \tag{2}$$

Initial condition

$$\phi_\gamma(\mathbf{r},0) = \phi_{\gamma 0}(\mathbf{r}) \tag{3}$$

Boundary conditions

$$-\mathbf{n}_{\gamma_\sigma} \cdot D_\gamma \nabla \phi_\gamma = -\mathbf{n}_{\gamma_\sigma} \cdot D_\sigma \nabla \phi_\sigma \qquad \text{at} \qquad \gamma\sigma\text{-interface} \tag{4}$$

$$\phi_\sigma = \phi_\sigma, \qquad \text{at} \qquad \gamma\sigma\text{-interface} \tag{5}$$

where φ is the neutron flux, **J** is the vector current density, Σ_a is the absorption cross section, S is the neutron source, D is the neutron diffusion coefficient and $\mathbf{n}_{\gamma\sigma}$ is the unit normal vector directed from the γ-phase towards the σ-phase.

In these equations

$$\Sigma_a(\mathbf{r}) = \Sigma_t(\mathbf{r}) - \Sigma_s(\mathbf{r}) \tag{6}$$

$$\Sigma_{tr}(\mathbf{r}) = \Sigma_t(\mathbf{r}) - \bar{\mu}_0 \Sigma_s(\mathbf{r}) \tag{7}$$