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The dynamic stress intensity factor around the anti-plane crack in an infinite strip functionally graded material under impact loading

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ABSTRACT

In order to dynamically analyze a crack in an infinite strip functionally graded material for anti-plane crack problem under impact loading, a gradient model is established in which the mass density and the shear modulus in the two principal directions of the functionally graded material are assumed to vary proportionally according to the double parameters index function. By using the Laplace and Fourier transforms, the motion equation is first reduced to the Euler equation. By solving dual integral equations derived by applying the solution of the Euler equation with the method of Copson, dynamic stress intensity factor around the crack tip is derived. The variation curves of the normalized stress intensity factor with the inhomogenous coefficient and gradient parameter are obtained from the numerical calculation. The results show that the decrease of the inhomogenous coefficient and the increase of the gradient parameter are beneficial to the decrease of the dynamic stress intensity factor.

1. Introduction

Functionally graded materials (FGM) are new advanced composites characterized by the gradual variation in composition, microstructure and material properties. The concept of FGM has been introduced to eliminate singular stresses, relax residual stresses, and enhance bonding strength. Thus, fracture analysis of FGM is an important topic in the optimization, design and novel engineering applications of FGM. FGM can be applied to a wide range of engineering structures and components such as wear-resistant coatings, biomaterials and electronic devices, thermal barrier coatings, optical films and corrosion-resistant. With the growing use of functionally graded materials (FGM) in engineering applications, the need for fundamental understanding of their mechanical behavior becomes imperative. The fracture and fatigue properties of FGM are important to their reliability, mechanical integrity, and durability. Cracks may be induced in functionally graded materials during the manufacturing process, as well as in-service loading conditions. Crack failure is one of the most dominant failure mechanisms in the materials. The knowledge of crack growth and propagation in functionally graded materials is important in designing components of FGM and improving their fracture toughness. Dynamic crack propagation in composite materials and the response of cracked composite bodies under dynamic loads is a subject that has been investigated, both theoretically and experimentally, to a large extent. In recent years, many studies have contributed to the fracture analysis of FGM. However, due to the mathematical difficulties arising from the fact that the properties of the FGM vary in space, most of the previous works were confined to some special cases. Sih [1–3] studied the dynamic stress intensity factors for a finite crack and a penny-shaped circular crack in an infinite medium subject to impact load. The stress intensity factors defined in the Laplace transform domain were inverted numerically to physical space using the numerical method. Atkinson [4] considered crack propagation in media with spatially varying elastic moduli, this variation being in a direction perpendicular to the crack growth direction. Erdogan et al. [5–8] studied the nature of stress singularity near the tip of a crack fully embedded in a relaxation, and the problem of stress concentration and the initiation and growth of delamination cracks from the stress-free ends of FGM-coated homogeneous substrates under residual or thermal stresses. In these studies the shear modulus of the nonhomogeneous layer was assumed to vary exponentially, and the Poisson's ratio kept constant. Babaei et al. [9,10] solved the transient dynamic problem for a crack in a nonhomogeneous layer between two dissimilar elastic half-planes. The material constants vary continuously in the layer. The crack surfaces are loaded suddenly by anti-plane shear traction, and the dynamic stress intensity factor for mode III loading is obtained.

Wang et al. [11–14] considered the axially symmetric problem of a cylindrical crack in the nonhomogeneous region of two coaxial

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Nomenclature

$2a$	length of the crack
$2h$	thickness of FGM strip
μ_x, μ_y	shear modulus
ρ	mass density
k, α	gradient parameter
u_x, u_y, u_z	displacement components in the directions x, y and z
$\omega(x, y, t)$	displacement function
τ_{xz}, τ_{yz}	stress components
$H(t)$	Heaviside unit step function
r	the inhomogeneity coefficient
τ^*_{xz}, τ^*_{yz}	stress components in the Laplace transform domain
$\omega^*(x, y, p)$	displacement function in the Laplace transform domain
$J_0(\cdot)$	zero Bessel function of the first kind
$K^*_{\mathrm{III}}(p)$	mode-III dynamic stress intensity factor in the Laplace transform domain
$K_{\mathrm{III}}(t)$	dynamic stress intensity factor

elastic cylinder under axially symmetric torsional loading. The material properties of the nonhomogeneous layer were assumed to power function. A fracture mechanics approach for a partially debonded cylindrical fiber was proposed by Brighenti et al. [15]. Jain et al. [16,17] employed the strain energy density criterion to obtain critical conditions for crack initiation in functionally graded materials with linearly varying properties, and found that materials with varying properties can offer more resistance to the crack propagation and suppress the crack growth in some situations. Li et al. [18–20] explored the fracture problem of functionally graded materials using the power-law model. Feng et al. [21,22] studied the fundamental solution of a power-law orthotropic and half-space functionally graded material under line loads. Huang and Wang [23,24] discussed a series of crack problems in FGM by assuming that the Young's modulus was a piecewise linear function of the coordinate y and the Possion's ratio was constant. Cheng and Zhong [25–27] studied the fracture of functionally graded materials under plane deformation by assuming that the reciprocal of the shear modulus was a linear function of the thickness-coordinate. Ma and Zhong [28] studied the plane elastic problem of a FGM strip with a crack by assuming that the shear modulus of FGM has a power form with the Possion's ratio kept constant.

In this paper, the models of double parameters index function are adopted to study the dynamic stress intensity factor around the anti-plane crack in an infinite strip functionally graded material under impact loading. By using the Laplace and Fourier transforms, the motion equation is first reduced to Euler equation. This method is different from the others. By solving dual integral equations that were derived by applying the solution of Euler equation with the method of Copson [29], the dynamic stress intensity factor around the crack tip is derived.

2. Formulation of the problem

Let us consider the anti-plane problem of an infinite strip functionally graded material containing a crack of length $2a$ as is shown in Fig. 1. The thickness of FGM strip is $2h$. Refer to a Cartesian coordinate system (o-xy), with the origin located at the crack center. The crack is subjected to an impact anti-plane crack-face loading. The double parameters index function model as a material property parameter model is adopted, shear modulus μ_x, μ_y and mass density ρ are functions of y, and μ_x, μ_y, ρ vary according to the following expressions:

$$\mu_x(y) = (\mu_x)_0 \exp(2k\alpha y) \tag{1a}$$
$$\mu_y(y) = (\mu_y)_0 \exp(2k\alpha y) \tag{1b}$$
$$\rho(y) = \rho_0 \exp(2k\alpha y) \tag{1c}$$

where $(\mu_x)_0$ and $(\mu_y)_0$ are the shear modulus at $y = 0$, k and α are called the gradient parameters describing the nonhomogeneity of FGM, and $k > 0$, $\alpha > 0$.

Suppose that u_x, u_y and u_z are displacement components of the directions x, y and z, respectively. For the anti-plane shear deformation, u_x, u_y are zero everywhere. Assume that $u_z = \omega\ (x, y, t)$, and $\omega(x, y, t)$ is the displacement function. The relationships between the displacement function $\omega(x, y, t)$ and stress components τ_{xz}, τ_{yz} are written as follows:

$$\tau_{xz} = \mu_x \frac{\partial \omega}{\partial x}, \quad \tau_{yz} = \mu_y \frac{\partial \omega}{\partial y} \tag{2}$$

The cracked FGM satisfies the equation of the stress equilibrium

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \rho \frac{\partial^2 \omega}{\partial t^2} \tag{3}$$

Substituting Eq. (2) into the stress equilibrium Eq. (3), the motion equation of anti-plane problem in infinite strip functionally graded materials can be obtained as follows:

$$\frac{\partial^2 \omega}{\partial x^2} + \frac{\mu_y}{\mu_x} \cdot \frac{\partial^2 \omega}{\partial y^2} + \frac{\mu_y'}{\mu_x} \frac{\partial \omega}{\partial y} = \frac{\rho}{\mu_x} \cdot \frac{\partial^2 \omega}{\partial t^2} \tag{4}$$

where μ_y' is the differential function of the shear modulus μ_y with respect to the coordinate y.

The corresponding boundary conditions of the crack surface under anti-plane impact shear loading τ_0 are as follows [1,11,28,30,31]:

$$\tau_{yz}(x, 0, t) = -\tau_0 H(t), \ |x| \leqslant a, t > 0 \tag{5.1}$$
$$\omega(x, 0, t) = 0, \ |x| > a, t > 0 \tag{5.2}$$
$$\tau_{xz}(x, \pm h, t) = 0, \ |x| < \infty, t > 0 \tag{5.3}$$

where $H(t)$ is the Heaviside unit step function.

3. Dual integral equation

Due to the symmetry, we only need to study the part where $x > 0$, $y > 0$. Introduce the Laplace transform as follows:

$$L[w(x, y, t)] = w^*(x, y, p) = \int_0^\infty w(x, y, t) \exp(-pt) dt \tag{6}$$

$$L^{-1}[w^*(x, y, p)] = w(x, y, t) = \frac{1}{2\pi i} \int_{Br} w^*(x, y, p) \exp(pt) dp \tag{7}$$

where Br denotes the Bromwich path of integration.

By using the Laplace transform, the time variable can be removed, and the motion Eq. (4) becomes

$$\frac{\partial^2 \omega^*}{\partial x^2} + \frac{\mu_y}{\mu_x} \cdot \frac{\partial^2 \omega^*}{\partial y^2} + \frac{\mu_y'}{\mu_x} \frac{\partial \omega^*}{\partial y} = \frac{\rho}{\mu_x} \cdot p^2 \omega^* \tag{8}$$

Application of the Laplace transform to the boundary conditions gives us: