Theoretical and Applied Fracture Mechanics 74 (2014) 39–47

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Numerical stochastic analysis of RC tension bar cracking due to restrained thermal loading

Jens U. Hartig *, Ulrich Häussler-Combe

Institute of Concrete Structures, Faculty of Civil Engineering, Technische Universität Dresden, 01062 Dresden, Germany

ARTICLE INFO

Article history:
Available online 1 July 2014

Keywords:
Reinforced concrete
Cracking
Thermal loading
Numerical model
Random field
Statistics

ABSTRACT

Cracking of concrete has deteriorating effects on the behavior of reinforced concrete structures, which results for instance in stiffness reduction and increase of permeability. In general, it is assumed that the deterioration increases with increasing crack width and, thus, maximum crack width might be most critical. However, models and recommendations in design codes regarding the serviceability limit state are usually based on mean crack width. In this contribution, a model for tensile members based on the Finite Element Method is presented, which takes spatially scattering material properties into account. As a special loading case restrained thermal loading is considered. The model is validated based on comparison with experimental results and with an analytical model. As an example of application, the model is embedded in a Monte-Carlo simulation to predict the statistics of crack widths due to spatial scatter of concrete tensile strength and bond strength. Results are crack width distributions and characteristic statistical values like mean values, standard deviations and quantile values over the entire loading range. Besides the presentation of the methodology of determining statistical properties of crack widths, it is shown that mean and maximum crack widths might be underestimated by analytical models as recommended in current design codes.

1. Introduction

Imposed deformations, e.g., due to temperature changes and concrete shrinkage, often result in concrete cracks in restrained reinforced concrete (RC) structural members, which might impair serviceability and structural integrity. Available design codes and recommendations as, e.g., CEB-FIP Model Code 90 [1], "Eurocode 2: Design of concrete structures" [2] and "ACI Building Code Requirements for Structural Concrete" [3], are rather vague in predicting cracking properties especially concerning restrained imposed loading. A more detailed description of this issue is given by the authors in a recent paper [4] where also an extension of the analytical model for the estimation of crack width as provided by [1,2] regarding imposed deformations is presented. While this analytical model is an efficient means for the estimation of mean crack widths, it does not account for stochastic variations of crack widths due to the scattering material properties. For design purposes, especially the maximum crack width values are of interest as they might have the most detrimental influence on the performance of tensile structural members.

In the present contribution, the statistics of crack widths in RC bars exposed to direct tension resulting from restrained imposed deformations are analyzed. The investigations are focused on the case of thermal loading caused e.g. by insolation or other environmental temperature changes. For the determination of statistical properties of crack widths, Monte-Carlo-type simulations are performed. This is based on a deterministic Finite Element model representing the load-bearing behavior of RC tension bars. The model consists of one-dimensional bar elements and zero-thickness bond elements. It incorporates nonlinear constitutive relations for concrete and reinforcement as well as the bond in between. Such a type of model was used in [5] for the analysis of the tensile behavior of Textile Reinforced Concrete. However, the statistical properties of crack widths were not analyzed. A similar model extended to 2D was applied by [6] for the analysis of continuously reinforced concrete pavements. Another 2D model, where the reinforcement contribution is taken into account in a so-called embedded crack FE model, was developed in [7]. Both models provide only mean values of crack width.

For the validation of the numerical model, experimental results from literature are used. Experimental results regarding cracking

* Corresponding author. Present address: Institute of Steel and Timber Construction, Faculty of Civil Engineering, Technische Universität Dresden, 01062 Dresden, Germany. Tel.: +49 351 463 35784.

E-mail addresses: jens.hartig@tu-dresden.de (J.U. Hartig), ulrich.haeussler-combe@tu-dresden.de (U. Häussler-Combe).

http://dx.doi.org/10.1016/j.tafmec.2014.06.004
0167-8442/

Nomenclature

$A_{c,eff}$	effective cross-sectional area of concrete
A_s	cross-sectional area of reinforcement
E	Young's modulus
E_c	Young's modulus of concrete
E_s	Young's modulus of reinforcement
F_{imp}	force due to restrained imposed loading
L	length of bar
$\mathbf{P}$	autocorrelation matrix
$\boldsymbol{S}$	realization of random field
T	temperature
ΔT	temperature change
Y	random variable
d_s	diameter of reinforcement bar
f_{ct}	tensile strength of concrete
f_{st}	tensile strength of reinforcement
f_{sy}	yield strength of reinforcement
l_t	stress transfer length
l_{corr}	correlation length
n	number of cracks
n_{rv}	number of random variables
n_{set}	number of simulations in MCS
s	slip
s_f	slip corresponding to τ_f
s_{max}	slip corresponding to τ_{max}
s_r	crack spacing
W	crack width
x	longitudinal coordinate
α_e	ratio between Young's moduli of reinforcement and concrete
α_T	thermal expansion coefficient
β_t	shape coefficient of reinforcement stress course
ε	strain
ε_c	concrete strain
$\varepsilon_{c,imp}$	imposed concrete strain
ε_{cm}	mean concrete strain
ε_E	imposed strain
ε_s	reinforcement strain
$\varepsilon_{s,imp}$	imposed reinforcement strain
ε_{sm}	mean reinforcement strain
λ	eigenvalue of $\boldsymbol{P}$
ξ	standard Gaussian random number
ρ_{eff}	effective reinforcement ratio
$\rho_{Y,Y'}$	autocorrelation coefficient
σ	stress
σ_c	concrete stress
σ_{cm}	mean concrete stress
σ_{cr}	concrete stress at the crack
σ_{imp}	eigenstress
σ_s	reinforcement stress
σ_{sm}	mean reinforcement stress
σ_{sr}	reinforcement stress at the crack
$\Delta\sigma_s$	difference between minimum and maximum reinforcement stress
τ	bond stress
τ_f	residual/frictional bond stress
τ_m	mean bond stress
τ_{max}	bond strength
$\boldsymbol{\psi}$	eigenvector of $\boldsymbol{P}$

of RC tension members due to thermal loading were performed in [8–11].

2. Modeling

2.1. Revision of analytical model in common design codes

In this section, the analytical model presented in [4] based on [2,1] is briefly summarized. The results of the numerical stochastic model will be subsequently compared to the prediction of this model.

The model distinguishes between single cracks and stabilized cracking. At a crack, differences in concrete strain ε_c and reinforcement strain ε_s occur along the stress transfer lengths l_t, see Fig. 1.

The crack width w is given as the integral of the differences between ε_c and ε_s along longitudinal direction x where the crack is situated at $x = 0$

$$w = 2\int_0^{l_t} [\varepsilon_s(x) - \varepsilon_c(x)]dx = 2l_t(\varepsilon_{sm} - \varepsilon_{cm}), \quad (1)$$

which can be simplified using the mean strains of the concrete ε_{cm} and the reinforcement ε_{sm} in the stress transfer length l_t. At a crack, the reinforcement stress has its maximum value, which is denoted with σ_{sr}. Neglecting the post-cracking resistance of the concrete, the concrete stress is zero at a crack. The value of σ_{sr} can be determined based on the normal force applied to the RC bar. The difference in the reinforcement stress between the maximum value at the crack σ_{sr} and the minimum value distant to the crack is denoted with $\Delta\sigma_s$. The mean value of the reinforcement stress along l_t is given with

$$\sigma_{sm} = \sigma_{sr} - \beta_t\Delta\sigma_s \quad (2)$$

where β_t is an empirical factor ($0 < \beta_t < 1$) depending on the shape of the reinforcement stress distribution along l_t. Values of β_t are provided for typical loading conditions in [2,1].

Forces are transferred between concrete and reinforcement by means of bond stresses τ leading to increasing tensile stresses in the concrete with increasing distance to the crack. The bond stress τ is controlled by the slip s, which is the relative displacement between the concrete and the reinforcement. The respective relation $\tau(s)$ is also called bond law. At a position x', the slip s is given as the integral

$$s(x') = \int_{x'}^{l_t} [\varepsilon_s(x) - \varepsilon_c(x)]dx. \quad (3)$$

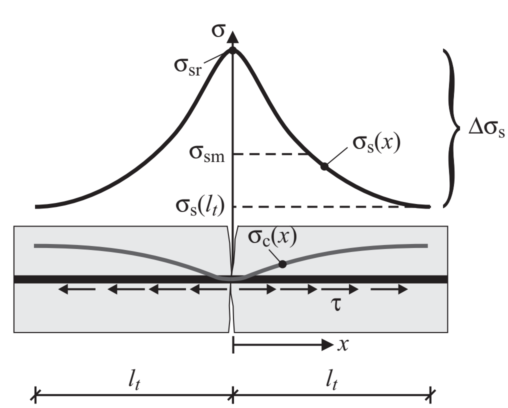

Fig. 1. Stress distribution in the stress transfer length.