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Analytical characterization of plastic flow in spot welded joints

Pierluigi Fanelli [a], Francesco Vivio [b,*]

[a] DEIM, University of Tuscia, Largo dell'Università, 01100 Viterbo, Italy

[b] Department of Enterprise Engineering, University of Rome ,"Tor Vergata", Via del Politecnico 1, 00133 Rome, Italy

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ABSTRACT

An analytical procedure for the evaluation of the elastic–plastic behaviour of spot welded joints, in terms of global stiffness is presented in this paper. The analytical procedure is based on a new reference theoretical model of the sheet region close to spot welded joint able to follow the evolution of plastic flow close to the joint. The new closed-form solutions allow to define the deflection of spot joint when an axial load is applied and plasticization occurred. The reference theoretical model is based on theory of elasticity and consists in a circular plate having two annular portions with different variable thickness and having a central rigid core representing the spot weld. This model allows to correctly simulate the actual plasticization radius when load increases and the global stiffness of the actual joint.

The analytical procedure presented is new and lead to the definition of a spot weld finite element, able to precisely evaluate both local and overall stiffness of spot joints, also in elastic plastic field. This spot weld element lead to accurate simulation of multi spot welded structures with a very low computational effort.

1. Introduction

The interest for structures jointed by means of spot welding is increasing, especially in its applications in the automotive and railway field, where even thousands of spot welds are present. These junctions are subjected to complex multi-axial loads in both operating and exceptional conditions. The correct evaluation of local stiffness of the spot weld and, more generally, of its structural behaviour requires an accurate modelling with finite elements of the region next to the spot, which is characterised by high stresses and local deformations; on the other hand, a reduced computational effort, in particular in the analysis of structures of actual interest with several spot welds (even thousands) is needed. Then the objective is an accurate evaluation of the local stiffness of spot joints, introducing adequate modelling of the region close to each spot joint, which is fundamental to perform a reliable simulation of multi-jointed structures and, consequently, a good estimate of loads acting on spots. A correct estimation of local stiffness, in both elastic and inelastic conditions, also leads to better introduce approaches for analysing local phenomena of interest such as criteria to predict fatigue behaviour of the joint and, consequently, of the complete structure.

In literature there are many studies on spot joints, focused on the assessment of fatigue life or on the evaluation of failure modes of spots in operating conditions, in elastic or elastic–plastic field. In these cases it is necessary to evaluate the structural behaviour of the joints and of the complete structure, in terms of stiffness, in order to calculate the loads that are exchanged between the welded joints and the rest of the structure and the stress state near to the welded joints.

In any case, when a complete spot welded structure is analysed, it is fundamental the correct evaluation of loads acting on spots in order to obtain reliable results from any local approach.

Fatigue life estimation has been investigated using different approaches. Generally, the main effort has been made to evaluate stresses at the joints, using complex FE models, theoretical approaches, experimental evidences or any combination of the above. Often, fracture mechanics has been used to evaluate stress intensity factors in natural crack or notch along the nugget circumference. In this case, various types of spot welded connections and different geometries of joint have been investigated using finite element models, considering linear elastic material behaviour [1,2] or elastic–plastic material behaviour [3–5], or introducing approximate or complete analytical solutions [6–8] for stress intensity factor. It should be noted that in these cases fatigue behaviour models are often only applicable and valid for reference joints of simple shape.

In other cases it has been proposed the use of a conventional stress parameter, demonstrating its effectiveness to predict spot weld fatigue life. In this context, simple analytical models, in linear elastic condition, can lead to a good estimate of the joint stiffness

* Corresponding author. Tel.: +39 06 72597123.
E-mail address: vivio@uniroma2.it (F. Vivio).

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and of the loads interchanged between the spot weld and the rest of the structure; these loads can be related to a conventional stress parameter at the spot weld using simple analytical/experimental methods [9] or theoretical models solved analytically [10–12].

Nevertheless, in literature there are not analytical–theoretical model able to characterise the elastic–plastic behaviour of the metal sheet jointed with spot welds. The complications due to the occurrence of plastic strain make the problem hard to approach theoretically and it is often necessary the use of complex FE models. However, when modelling actual structures with several spot welds, it is essential to make use of drastic simplifications, which have a considerable impact on results.

In this context multiple attempts have been made in this direction in the past, by introducing various degrees of simplification of the geometry, modelling the spot weld with individual beam elements [13,14]. This approach, which is currently the preferential way to deal with the simulation of the overall behaviour of structures with several joints, involves a loss of accuracy, especially in relation to the definition of the local stiffness and of the distribution of loads on spot welds.

To overcome such limitations, it is possible to analyse the behaviour of a spot weld joint using the theory of bending of thin plates, in a similar way to what was done for joints stressed in the elastic field [10], but in elastic–plastic condition.

However, in literature, high interest is aimed on the evaluation of the total plasticity of the sheet metal and on the evaluation of limit conditions of the joint, but not on the accurate analysis of the evolution of plastic flow and the correlate variation of local stiffness of the joint and of the energy absorption. Solutions for the problem of plastic collapse of a circular plate in plastic field have been obtained in the past by numerous authors (for example [15]). These models are related only to the final state of the plate and do not allow the evaluation of the progressive collapse of the structure. In only a few cases analytical methods able to analyse the evolution of plastic flow have been introduced (from the condition of incipient plasticity to collapse), but the considering very restrictive assumptions (on material behaviour, on kinematic failure modes and on resistance criterion), with unavoidable impact on accuracy of results. The introduction of these assumptions can allow to solve the theoretical models, in some cases with simplified closed-form solutions, often not supported by experimental results. Skrzypek and Hetnarski [16] introduced an analytical approach to solve the circular plate stressed over the elastic limit, in small displacement hypothesis: despite the restrictions introduced, the complex system of differential equations leads to a simplified solution (not in a closed-form) that is approximated and far from the actual behaviour.

Faced with the difficulties in obtaining an accurate analytical solution, due to the high non-linearity of the equation system, several numerical approaches were proposed [17,18]. The most cumbersome aspects are represented by the definition of the plastic flow through the plate thickness and the implementation of membrane effects occurring when large deflections are introduced and Kirchhoff–Love hypothesis are removed.

For the analysis of elastic–plastic behaviour of these joints a rather different approach has been proposed in [19]; this is based on an analytical procedure able to evaluate the behaviour of the spot welded joints, in case of large displacements and beyond yielding, by means of the introduction of an equivalent circular plate model (a plate with linearly variable thickness along the radius and with a clamped central rigid inclusion) solved in linear elastic conditions. Through the elastic analysis of this reference theoretical model it is possible to obtain the closed-form solution, in terms of displacement of the central nugget of the actual spot weld joint, in presence of plasticity and large displacements.

These results are the basis of the enhancement of the spot weld element (whose complete theoretical definition and validation, in elastic field, has been presented in [20]) in elastic–plastic field. This spot weld element is composed by two complex sets of equivalent one-dimensional elements, simulating the two portions of metal sheet, connected by a rigid link. The definition of the non-linear stiffness of the link (instead of the rigid link), using the non-linear solution of the reference theoretical model of the joint (the equivalent circular plate with linearly variable thickness), allows the use of the spot weld element also beyond the elastic limit. In [21] the authors investigated the structural behaviour of the reference theoretical model defined in [19] when different constraint conditions are imposed to the outer radius. In particular, it has been demonstrated that the global decrease of joint stiffness, that occurs beyond material yielding, can be modelled as a system of two in-series equivalent stiffness (this stiffness system may constitute the equivalent stiffness of the spot weld element link): the former simulates the stiffening effect of large displacements while the latter simulates the weakening effect due to plasticization (obviously higher than the former). In this work it has also been demonstrated that the weakening effect due to plasticization of the plate is independent from the boundary conditions, despite the influence of the constraint conditions on the configuration of plasticization fronts. Such evidence allows to correctly translating the analytical solution of the reference theoretical model (which is define with a hinging at the outer radius) in a spot weld element (to be considered as a finite element).

In this paper, a variation of the reference theoretical model introduced in [19] is proposed and analytically solved, in closed-from, in linear elastic conditions. The new characteristic of the reference model consists in the subdivision of the plate in three portions having different laws for thickness variation (the previous reference theoretical model had two portion having different thickness variation). This improvement of the model is derived from an accurate numerical analysis of the actual behaviour of thin circular plate properly constrained and loaded orthogonally, as described in [19,21], whose geometrical characteristics reproduce portion of metal sheets typically joined with spot welds in consolidated applications in mechanical structures of engineering interest. Refined FE models of actual circular plate have been used to accurately analyse the evolution of plasticization flow when the applied load increases.

The closed-form solutions of the new reference theoretical model are based on the analytical approach proposed in [22] for bending circular plate having general non-linear variable thickness. The Gauss differential equation of second order with non-constant coefficients, describing the rotation field in each sector of the circular plates, is solved introducing two independent integrals with hypergeometric functions.

The new reference theoretical model allows to characterise more accurately the actual profile of plasticized material and its evolution with the increase of the applied load. The analytical evaluation of the non-linear stiffness of the spot joint beyond the elastic limit is the basis for the extension of the formulation of a spot weld element, by means of a new definition of non-linear stiffness of its central link. In this way it is possible to simulate, in elastic plastic field, multi spot welded structures with simplified FE models, with great accuracy of evaluation of the local stiffness of each spot joint and with a very low computational effort.

2. Preliminary numerical analyses

The reference theoretical model proposed in [19] has been defined considering the simulation of the front of plasticization that, in a diametrical section of the plate, has a profile that can