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A micromechanical-based damage analysis of a cylindrical bar under torsion: Theoretical results, Finite Elements verification and application

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ABSTRACT

We first present exact closed-form solutions to the problem of a cylindrical bar subjected to a torsion loading. The bar is made of a material whose behavior is modeled by means of a class of micromechanically based isotropic elastic damage models. It is shown that under an increasing torsion, the bar exhibits a global softening regime related to its progressive deterioration. The paper also provides explicit expressions of the mechanical fields as well as of the damage distribution in the bar. A careful attention is given to the response during an unloading step. Finally, after implementing the damage models in a Finite Element software, we simulate the bar response under the same torsion loading. Interestingly a full agreement is noted between the theoretical predictions and the numerical results, also in the softening regime. Finally, the proposed models are applied to a gray cast iron; the predictions compare well to experimental data.

1. Introduction

Civil engineering structures generally undergo damage phenomena by microcracks growth when they are subjected to mechanical loadings (see for example [8,17]). The design of such structures requires an accurate determination of the displacements, stresses and damage fields; this strongly depends on the considered constitutive law. For structures made of concrete or rock-like materials, application of damage models is an essential step in order to satisfactorily predict the behavior up to failure. However, due to the occurrence of materials softening, this represents a real scientific challenge in so far as it appears as a limitation for the transfer of damage models in engineering practice. The main objective of the present study is double:

(i) To establish closed-form solutions for a structural problem in the context of quasi-brittle materials sustaining damage; it is expected that such solutions can be used as a reference for academic benchmarks. To this end, we are interested by the response of a cylindrical bar, the mechanical behavior of the considered concrete material being described by means of a class of micromechanical-based damage models.

(ii) To provide a first verification of the established solutions by performing Finite Elements calculations on the basis of the considered class of damage models. The computations are carried out after a careful implementation of the models in Abaqus software via an UMAT Routine.

The paper is organized as follows. First, the basic elements of the considered class of isotropic damage models are briefly presented.[1] The models are based on the consideration of a scalar internal damage variable and physically rely on results provided by linear homogenization schemes applied to microcracked materials; this class of micromechanically-inspired damage models has been recently proposed in the context of description of the unilateral effects (see [7,1]). Then, we solve the problem of a cylindrical bar made up of an elastic damage material (a concrete for instance) and subjected to a torsion load on the external boundary. The solutions are presented and comparatively analyzed and discussed for the different schemes (details of some solutions are given in appendix). A particular emphasis is put on unloading phases in order to clearly show the difference with a perfectly plastic material. The last part proposes a numerical assessment of the established analytical results. After numerically implementing the damage models, this is done by performing Finite Elements calculations on the cylindrical

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[1] This is completed by an appendix in which are detailed the components of the microcracks-induced damage models, issued from different homogenization schemes.

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bar. Up to a critical mechanical state, well identified for the considered damage models, the comparisons showed (for the stress field, the damage field, and for the torque-twist response) a very good agreement between the theoretical results and the FE. The last section is devoted to an application of the models to a gray cast iron for which uniaxial tension data are used for parameters identification. Comparison of the predictions for pure torsion to experimental data indicates a very good performance of the model based on the Mori–Tanaka scheme.

2. Presentation of damage model: thermodynamics-based formulation

To describe the degradation of mechanical properties in quasi-brittle materials (for instance concrete or some rocks), let us consider a class of isotropic and micromechanically-inspired damage models.

In this framework, isothermal small perturbations without viscous effects are assumed and the unique dissipation mechanism taken into account is damage by microcracking. Owing to the assumption of an isotropic damage, the latter is characterized by a single scalar internal variable d. This variable is positive valued and continuously increasing and can be interpreted as a micro-cracks density parameter (see [2]).

2.1. Thermodynamics potential and state laws

At constant damage d, the mechanical behavior of the damaged material is considered elastic and isotropic. The elastic law, dependent on the damage d, is characterized by the free energy density function $\psi(\underline{\underline{\varepsilon}},d)$ where $\underline{\underline{\varepsilon}}$ denotes the strain tensor. The free energy function ψ acts as the thermodynamics potential of the elastic material and reads:

$$\psi(\underline{\underline{\varepsilon}},d)=\frac{1}{2}\underline{\underline{\varepsilon}}:\mathbb{C}(d):\underline{\underline{\varepsilon}}=\frac{1}{2}k(d)\mathrm{Tr}\left(\underline{\underline{\varepsilon}}\right)^2+\mu(d)\underline{\underline{\varepsilon}}^d:\underline{\underline{\varepsilon}}^d \quad (1)$$

In which $\mathbb{C}(d)$ has been considered as fourth order isotropic stiffness tensor of the elastic damaged material: $\mathbb{C}(d)=3k(d)\mathbb{J}+2\mu(d)\mathbb{K}$, with tensor $\mathbb{J}=\frac{1}{3}\underline{1}\otimes\underline{1}$ and tensor $\mathbb{K}=\mathbb{I}-\mathbb{J}$. Note that the identity second order tensor and symmetric fourth order tensor are respectively defined by their following components: $1_{ij}=\delta_{ij}$ and $I_{ijkl}=\frac{1}{2}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})$.

In (1), $k(d)$ and $\mu(d)$ are respectively the bulk and the shear moduli of the damaged material and decrease when the damage variable $d$increases.$\underline{\underline{\varepsilon}}^d$ denotes the deviatoric part of the strain $\underline{\underline{\varepsilon}}$. It is worth noting that for the determination of the two functions, different homogenization schemes will be considered later. The corresponding expressions will be provided in Sections 3.3 and 3.4.

By derivating the free energy, one obtains the stress tensor:

$$\underline{\underline{\sigma}}\left(\underline{\underline{\varepsilon}},d\right)=\frac{\partial\psi}{\partial\underline{\underline{\varepsilon}}}\left(\underline{\underline{\varepsilon}},d\right)=\mathbb{C}(d):\underline{\underline{\varepsilon}}=k(d)\mathrm{Tr}\left(\underline{\underline{\varepsilon}}\right)\underline{1}+2\mu(d)\underline{\underline{\varepsilon}}^d \quad (2)$$

The thermodynamics force $F^d(\underline{\underline{\varepsilon}},d)$ associated to the damage variable d, the so-called damage energy release-rate, reads:

$$\begin{aligned}F^d\left(\underline{\underline{\varepsilon}},d\right)&=-\frac{\partial\psi}{\partial d}\left(\underline{\underline{\varepsilon}},d\right)=-\frac{1}{2}\underline{\underline{\varepsilon}}:\mathbb{C}'(d):\underline{\underline{\varepsilon}}\\&=-\frac{1}{2}k'(d)\mathrm{Tr}\left(\underline{\underline{\varepsilon}}\right)^2-\mu'(d)\underline{\underline{\varepsilon}}^d:\underline{\underline{\varepsilon}}^d\end{aligned} \quad (3)$$

where $\mathbb{C}'(d)=3k'(d)\mathbb{J}+2\mu'(d)\mathbb{K}$ with $k'=\frac{\partial k}{\partial d}$ and $\mu'=\frac{\partial\mu}{\partial d}$.

2.2. Damage yield function and damage evolution law

The evolution of the damage variable d is assumed to be governed by a standard law with a damage threshold denoted G_c. To this end, the damage criterion is expressed in the following form:

$$f(F^d)=F^d-G_c\leqslant 0 \quad (4)$$

G_c is a constant positive valued scalar, and then defines also the current damage threshold. The thermodynamics force $F^d(\underline{\underline{\varepsilon}},d)$, defined by (3), plays the role of the damage energy release rate.

Finally, the complete formulation of the perfectly elastic damage model requires the formulation of the two constitutive functions: $d\mapsto k(d)$ and $d\mapsto\mu(d)$ to characterize the degradation of elastic properties. This can be done by taking advantage of available experimental data or by means of appropriate results from homogenization of microcracked materials. In the latter case, different schemes or bounds can be considered (see for example, [12,6,16,5,4]); their main helpful results for the present study are briefly recalled in Appendix A. The reader interested in the homogenization is referred for instance to Refs. [18,3,13,14].

For completeness, we provide here the damage evolution law obtained by assuming a normality rule:

$$\dot{d}=\begin{cases}-\dfrac{\dfrac{\partial F^d}{\partial\underline{\underline{\varepsilon}}}:\dot{\underline{\underline{\varepsilon}}}}{\dfrac{\partial F^d}{\partial d}} & \text{if } f(F^d)=0,\dot{f}(F^d)=0\\ 0 & \text{if } f(F^d)\leqslant 0,\dot{f}(F^d)<0\end{cases} \quad (5)$$

with

$$\frac{\partial F^d}{\partial\underline{\underline{\varepsilon}}}\left(\underline{\underline{\varepsilon}},d\right)=-k'(d)\mathrm{Tr}\left(\underline{\underline{\varepsilon}}\right)\underline{1}-2\mu'(d)\underline{\underline{\varepsilon}}^d$$

and

$$\frac{\partial F^d}{\partial d}\left(\underline{\underline{\varepsilon}},d\right)=-\frac{1}{2}k''(d)\mathrm{Tr}\left(\underline{\underline{\varepsilon}}\right)^2-\mu''(d)\underline{\underline{\varepsilon}}^d:\underline{\underline{\varepsilon}}^d$$

where $k''(d)=\frac{\partial k'}{\partial d}$ and $\mu''(d)=\frac{\partial\mu'}{\partial d}$.

Note that the choice of this kind of damage criterion and evolution law (Eq. (4) together with (5)) can be justified from the Drucker-Ilyushin postulate (see for instance [11]).

The differentiation of the state law (2), combined with the damage evolution law (5), readily yields the rate formulation of the elastic damage constitutive.

3. Theoretical study of a cylindrical barsustaining damage under torsion

3.1. Preliminary

We are now interested with the study of a cylindrical bar having a circular cross section of radius R and a height h. This bar is subjected to an increasing torsion (cf. Fig. 1).

Its lower base is laterally fixed while its upper base is subjected to a twist angle α; no effort along $\underline{e}_z$ is applied on these bases. Body forces are neglected and the outer edge of the bar is free of loading. The displacement $\underline{u}$ and the stress vector $\underline{T}$ boundary conditions then read:

$$\begin{aligned}&z=0: \quad u_r=0,\quad u_\theta=0,\quad T_z=0\\&z=h: \quad u_r=0,\quad u_\theta=\alpha r,\quad T_z=0\\&r=R: \quad 0<z<h,\quad \underline{T}=\underline{0}\end{aligned} \quad (6)$$

The bar is assumed undamaged in its initial state for which $\alpha=0$ and behaves linearly.