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Investigation of the behavior of a mixed-mode crack in a functionally graded magneto–electro-elastic material by use of the non-local theory

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ABSTRACT

In this paper, we consider the problem of a mixed-mode crack embedded in an infinite medium made of a functionally graded magneto–electro-elastic material (FGMEEM) with the crack surfaces subjected to magneto–electro-mechanical loadings. Eringen's non-local theory of elasticity is applied to obtain the governing magneto–electro-elastic equations. To make the analysis tractable, it is assumed that the magneto–electro-elastic material properties vary exponentially along a perpendicular plane to the crack. Using Fourier transform, the resulting mixed-boundary value problem is converted into four integral equations, in which the unknown variables are the jumps of mechanical displacements, electric and magnetic potentials across the crack surfaces. To solve the integral equations, the jumps of displacements and electric and magnetic potential across crack surfaces are directly expanded in a series of Jacobi polynomials and the resulting equations are solved using the Schmidt method. Unlike classical magnetic, electric and elasticity solutions, it is found that no mechanical stress, electric displacement and magnetic flux singularities are present at the crack tips. This enables the use of the maximum stress as a fracture criterion. The primary objective of this study is to investigate the effects of crack length, material gradient parameter describing functionally graded materials and lattice parameter on the mechanical stress, magnetic flux and electric displacement field near crack tips.

1. Introduction

Smart materials exhibiting magneto-electric (ME) behavior as a novel product property of its constituent phases has stimulated the interest of researchers in recent years [17]. The ME phenomenon, predicted as early as 1894, is a coupled or two-field effect in which application of either an external magnetic or electric field elicits both electric polarization and magnetization [30]. By combining a piezoelectric and piezomagnetic phase, this effect can be realized in magneto-electric–elastic composites, with the coupled electric and magnetic response being a consequence of elastic interaction between the constituents [30]. Individually the constituent phases are incapable of inducing ME effect. However, when combined into a composite material, ME coupling arises as a new material property, in some cases resulting in a hundred times increase compared to a single-phase magnetic or electric material [43,34]. There is a continuing surge in research on these materials attributed to their tremendous potential for a wide range of engineering applications like magnetic field probes, electric packaging, acoustic, hydrophones, medical ultrasonic imaging, sensors, and actuators that are capable of converting magneto-electric energy into mechanical energy and vice versa [34].

The last two decades have witnessed investigators exploring the possibility of using functionally graded materials (FGMs) as a promising alternative to conventional homogenous coatings [5]. FGMs comprise of at least two-phase inhomogeneous particulate composites and are synthesized in such a way that the volume fractions of the constituents vary continuously along any desired spatial direction, resulting in materials having smooth variation of mechanical properties. Such enhancements in properties endow FGMs with qualities such as resilience to fracture through reduction in propensity for stress concentration. It is thus only natural to extend the concept of FGMs to magneto–electro-elastic materials to create a new class of functionally graded magneto–electro-elastic materials (FGMEEMs) with improved reliability. However, in general, these materials are known to be brittle and

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defects inevitably form during manufacturing and subsequent handling leading to possible failure under magneto–electro-mechanical service loads [43]. Therefore, investigating fracture behavior of such composites together with its influence on the coupled response is of paramount importance.

A survey of literature reveals that almost all previous investigators studied FGMEEM fracture problems employing classical continuum mechanics methods based on the local elasticity theory. Also, noteworthy is the fact that a significant number of these studies are devoted to analysis of anti-plane mode III crack problems. Few of these are succinctly summarized in the following paragraph.

Considering both magneto-electrically impermeable and permeable crack surfaces, Ma et al. [27] analyzed a functionally graded magneto–electro-elastic strip with a mode III crack. Adopting the same mechanical and magneto-electric loading conditions, with crack surfaces again assumed to be impermeable or permeable for the magnetic and electric fields, Ma et al. [28] studied the mode III surface crack problem in a functionally graded magneto–electro-elastic coating bonded to a homogeneous elastic substrate. Zhou and Wang [40] examined the mode III crack problem of two parallel symmetrical permeable cracks in functionally graded magneto-electric materials under anti-plane shear loading. Feng and Su [15] analyzed the dynamic behavior of magneto-electrically impermeable and permeable mode III cracks in functionally graded magneto–electro-elastic plates. In another study, Feng and Su [16] studied the dynamic problem of a mode III magneto-electrically impermeable internal or edge crack parallel to the graded direction in a FGMEEM strip. Li and Lee [24] considered the anti-plane mode III problem of a permeable crack intersecting the interface between two FGMEEM layers. Li and Lee [25] analyzed the anti-plane mode III problem of a magneto-electrically permeable crack at the interface between two bonded FGMEEM strips. Guo et al. [20] examined the anti-plane problem of a mode III crack in a bonded FGMEEM strip sandwiched between two functionally graded strips considering both magneto-electrically impermeable and permeable crack faces.

Unlike mode III crack problems in FGMEEMs, it is observed that studies considering mode I or mixed mode crack problems are limited. Zhou and Chen [39] solved the mode I crack problem in an FGMEEM infinite medium assuming air permeability within the crack. Most recently, Rekik et al. [31] analyzed the mixed-mode magneto-electrically impermeable crack embedded in a functionally graded magneto–electro-elastic infinite medium.

In all of the studies cited above, classical continuum mechanics methods based on the local assumption were used. According to local elasticity theory, the state of stress at a specific point in the material depends only on the state of strain at the same point. A major shortcoming of these studies is the presence of stress singularities at the crack tips not warranted by physical considerations.

In the past four decades, nonlocal elasticity theory is becoming the method of choice to study crack problems as it can overcome the major issues posed by the classical method. Nonlocal continuum mechanics initiated by Eringen [12,13] is based on the nonlocal elasticity model, where the state of stress at a given point is a function of the strain states at all points in the material. In contrast to classical local theory, this theory takes into consideration long range forces between molecules, and an internal length scale enters the constitutive relations as a material parameter. The nonlocal theory was employed to great success by Eringen [8–10] to investigate the stress near the tip of a sharp line crack in an isotropic elastic plate. In direct contrast to classical (or local) elasticity theory, it was shown that the stress field calculated using nonlocal theory did not contain singularities at the crack tips. Thus this enables the use of maximum stress criterion for prediction of fracture in a natural way.

The ability of nonlocal continuum methods to compute maximum stress circumventing singularities at the crack tips, prompted investigators to employ nonlocal theory to analyze cracks in smart structures. Fracture problems in piezoelectric materials [41,42] were studied using the non-local theory and as expected the solutions did not contain any stress and electric displacement singularities at the crack tips. In the past three years, a couple of studies employing nonlocal theory to investigate crack problems in magneto–electro-elastic materials have appeared in literature. Zhang et al. [36] used nonlocal theory to study magneto-electrically permeable mode I cracks in a piezoelectric/piezomagnetic composite material. Most recently, the same technique was used by Zhou et al. [45] to investigate two collinear limited-permeable mode I cracks in the same material. All these investigations clearly indicate the advantage of using nonlocal theory as it results in finite stress, electric displacement and magnetic flux in the magneto–electro-elastic medium which is in direct contrast to classical solutions.

Mathematical difficulties are the limitation when it comes to studies related to crack problems in functionally graded magneto–electro-elastic materials by the non-local theory. To the best of our knowledge, the magneto–electro-elastic behavior of functionally graded magneto–electro-elastic materials with a mixed mode-magneto-electrically impermeable crack has not been studied using the non-local theory in the open literature. In this paper, the concepts of the non-local theory are extended to solve the mixed-mode crack problem in a functionally graded magneto–electro-elastic material. The stress, electric and magnetic fields near the mixed-mode crack tips in functionally graded magneto–electro-elastic materials are investigated by use of the non-local theory with Schmidt method [29,35].

This paper is organized as follows. The formulation and the boundary conditions of the problem are presented in Section 2. The basic solution of the mixed-mode problem in terms of displacement, electric and magnetic fields as well as the solution of the system of integral equations using the Schmidt method are detailed in Section 3. Section 4 contains the validation of results, parametric study, observations and a discussion of fracture criterion based on the present work. Finally, concluding remarks are provided in Section 5.

2. Problem description and formulation

As shown in Fig. 1, the problem under consideration consists of an infinite graded magneto–electro-elastic medium described in the Cartesian (x,z) coordinate system. The medium is unbounded in both the x and z directions. The graded medium contains an embedded crack of length $2l$ located in the infinite medium along

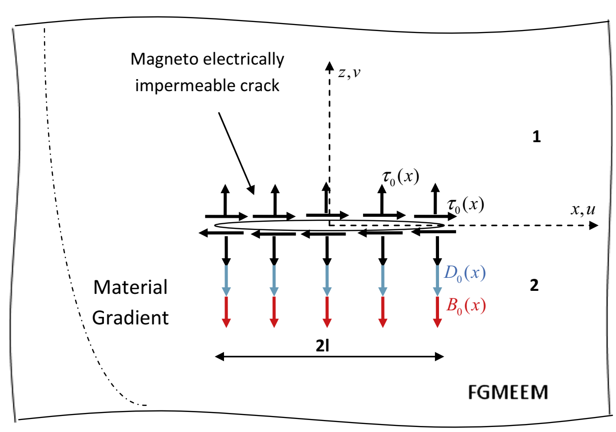

Fig. 1. Geometry and loading of the mixed-mode crack problem.