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Moving Dugdale type crack along the interface of two dissimilar piezoelectric materials

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ABSTRACT

In this paper we extend the concept of Dugdale crack model and Yoffe model to propose a moving Dugdale interfacial crack model, and the interfacial crack between dissimilar piezoelectric materials under anti-plane electro-mechanical loading is investigated considering the electro-mechanical nonlinearity. It is assumed that the constant moving crack is electrically permeable and the length of the crack keeps constant. Fourier transform is applied to reduce the mixed boundary value problem of the crack to dual integral equations, which are solved exactly. The explicit expression of the yield zone size is derived and the crack sliding displacement has been explicitly obtained. The results show that both the stress and electric field in the cracked piezoelectric material are of finite value and the crack sliding displacement is dependent on the loading, material properties and crack moving velocity. The static interfacial crack problem can be recovered when the moving velocity is zero.

1. Introduction

Because of the electromechanical coupling effect, piezoelectric materials have been widely used as transducers, sensors and actuators. Due to the brittleness and low fracture toughness of piezoelectric materials, fracture analysis of piezoelectric materials has drawn considerable attentions. Structural reliability concerns of electromechanical devices call for a better understanding of the mechanisms of piezoelectric fracture. Important results about fracture in piezoelectric solids based on linear electro-elasticity have been derived by Parton [1], Suo et al. [2], and many others. Theoretical investigation of crack propagation in elastic materials began with Yoffe's [3] analysis of the near-tip field of a constant moving crack, and some of the subsequent research works were carried out by Craggs [4], Freund [5], Yang et al. [6], among others. Considering the coupling effect of mechanical and electrical fields, the moving crack problem in piezoelectric materials has been studied by many researchers. The problem of a Griffith crack moving along the interface of two dissimilar piezoelectric materials was solved by Chen et al. [7] and Li et al. [8] using the integral transform technique, which showed that the stress distribution varies with the crack moving velocity while the stress intensity is independent of the velocity. A moving conducting crack at the interface of two dissimilar piezoelectric materials was investigated by Wang et al. [9]. The problems of moving crack in functionally graded piezoelectric materials under anti-plane shear and in-plane electric loadings have been investigated by Jin and Zhong [10], Hu and Zhong [11]. Soh et al. [12] investigated the general plane problem of constant moving crack in anisotropic piezoelectric materials and concluded that the crack velocity affects the crack tip fields and the propagation orientation of the moving crack. Hu and Chen [13] investigated the crack kinking phenomena in a piezoelectric strip under impact loading and studied the effect of geometric size and loading on the crack kinking. However, analyses based on the theory of linear electro-elasticity cannot explain some discrepancies between theories and experiments [14]. The linear electro-elastic fracture mechanics predicts that the stress and electric displacement at the tip of a Griffith crack is singular, which is physically unrealistic.

A moving Dugdale crack model has been proposed by Fan [15] who verified that dynamic crack opening, sliding and tearing displacements are significant for describing the dynamic process of materials with nonlinear behavior. Various nonlinear models have been suggested to study the crack problems in piezoelectric material. Gao et al. [16] generalized the essential idea of Dugdale [17] and proposed a strip yield saturation model of electrical yielding by assuming the electrical polarization is saturated in a line segment in front of the crack tips. Narita and Shindo [18] considered a mode III fatigue crack in a piezoelectric ceramic strip based on

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the Dugdale's model of the plastic zone. A strip dielectric breakdown model has been proposed by Zhang et al. [19] to analyze an electrically impermeable crack in piezoelectric medium. Shen et al. [20] developed a strip electric saturation and mechanical yielding model for a mode III interfacial moving crack between ferroelectric-plastic bimaterials. An analytical characterization of electromechanical nonlinear effects in the fields around the interfacial crack in piezoelectric material compound has been studied by introducing a pre-fracture zone model [21]. A moving polarization saturation model was proposed by Chen et al. to study the plane problem of a Yoffe-type crack moving in ferroelectric considering electric saturation [22].

In this paper, a Dugdale type interfacial crack between dissimilar piezoelectric materials under anti-plane shear and in-plane electrical loadings is studied. Fourier transform is employed to reduce the mixed boundary value problem of the electrically permeable crack to solving a pair of dual integral equations and the exact solution is obtained. The relation between the yield zone size and the applied loading is derived and the crack sliding displacement has been obtained.

2. Problem statement and basic equations

A way of removing the crack tip singularity was proposed in Dugdale [17], and according to this model, the assumption of a strip yield zone ahead of the physical crack tip is introduced; the plastic yielding is handled in an approximate manner by placing constraining yield level stress along a slit ahead of the crack tip in a manner which ensures bounded and continuous stresses at the edge of the plastic zone. Consider a Dugdale type interfacial crack of length $2c$ between dissimilar piezoelectric materials in a rectangular coordinate system (X, Y, Z) under anti-plane mechanical and in-plane electric loading at infinity, as shown in Fig. 1. The upper and lower part piezoelectric half-spaces are denoted by **I** and **II**, respectively. A finite shear yield stress σ_Y is prescribed along the yield zones described by $c \leqslant |x| \leqslant a$, and the length of the yield zone can be defined as $b = a - c$.

As in Yoffe's model [3,15], it is assumed that the interfacial crack is moving along the interface with a constant velocity v and the crack length remains unchanged. It seems that the assumption of constant length of the moving crack may not be realistic, since it demands the crack propagate at one end and healed at the other. However, the solution based on the Yoffe model [3] shows most of the features pertinent to dynamically moving crack in general, particularly the stress–strain field in the vicinity of a moving crack edge, and gives an explanation of limiting speeds and crack branching [23]. A set of moving Cartesian coordinates (x, y, z) is attached to the crack center for reference purpose. The piezoelectric materials are poled in the direction of Z–axis, which guarantees transverse isotropy of the piezoelectricity. Symmetry arguments are used to allow consideration of only the half space ($|x| \geqslant 0$) with appropriate boundary conditions along the coordinate axes.

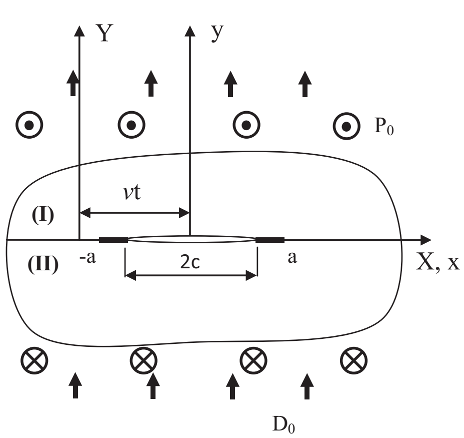

Fig. 1. A Dugdale interfacial crack moving along the interface between dissimilar piezoelectric materials.

Anti-plane displacement and in-plane electric field are assumed such that the constitutive equations can be written as

$$\begin{aligned} \sigma_{zj}^{(i)} &= C_{44}^{(i)} w_{,j}^{(i)} + e_{15}^{(i)} \phi_{,j}^{(i)} \\ D_j^{(i)} &= e_{15}^{(i)} w_{,j}^{(i)} - \lambda_{11}^{(i)} \phi_{,j}^{(i)} \end{aligned} \tag{1}$$

where $\sigma_{zj}^{(i)} (j = X, Y, i = 1, 2)$ are the shear stress components, $w^{(i)}$ is the z-component of displacement, $D_j^{(i)}$ are the components of electric displacement, $\phi^{(i)}$ is the electric potential, $C_{44}^{(i)}$ is the elastic stiffness constant measured in a constant electric field, $e_{15}^{(i)}$ is the piezoelectric constant, $\lambda_{11}^{(i)}$ is the dielectric constant measured at a constant strain, and a comma implies partial differentiation with respect to the coordinates. The superscript $i = 1, 2$ denotes the quantities in the upper and lower piezoelectric spaces, respectively.

In the absence of body forces and electric charge density, the governing equations are

$$\begin{aligned} & C_{44}^{(i)} \nabla^2 w^{(i)} + e_{15}^{(i)} \nabla^2 \phi^{(i)} = \rho^{(i)} \frac{\partial^2 w^{(i)}}{\partial t^2} \\ & e_{15}^{(i)} \nabla^2 w^{(i)} - \lambda_{11}^{(i)} \nabla^2 \phi^{(i)} = 0 \end{aligned} \tag{2}$$

where $\nabla^2 = \partial^2/\partial X^2 + \partial^2/\partial Y^2$ is the two-dimensional Laplace operator in the variables X and Y, $\rho^{(i)}$ is the mass density of the piezoelectric material. The governing equations can be further simplified and expressed as

$$\nabla^2 w^{(i)} = \frac{1}{V_C^{(i)^2}} \frac{\partial^2 w^{(i)}}{\partial t^2} \tag{3-1}$$

$$\nabla^2 \phi^{(i)} = \frac{e_{15}^{(i)}}{\lambda_{11}^{(i)}} \nabla^2 w^{(i)} \tag{3-2}$$

where

$$V_C^{(i)} = \sqrt{\frac{\mu^{(i)}}{\rho^{(i)}}}, \quad \mu^{(i)} = C_{44}^{(i)} + \frac{e_{15}^{(i)^2}}{\lambda_{11}^{(i)}}, \quad (i = 1, 2) \tag{4}$$

and $V_C^{(i)} (i = 1, 2)$ are the piezoelectric shear wave speeds in the upper and lower half-spaces, respectively, and $\mu^{(i)}$ $(i = 1, 2)$ are the piezoelectric stiffened elastic constants.

For the problem of a crack moving with a constant velocity along the X-direction, it is convenient to introduce a Galilean transformation as

$$x = X - vt, \quad y = Y \tag{5}$$

and one can obtain $\partial/\partial X = \partial/\partial x$, $\partial/\partial t = -v\partial/\partial x$, $\partial/\partial Y = \partial/\partial y$. Thus, in the moving coordinate system, Eqs. (3) become independent of the time variable t, and may be written as

$$\alpha^{(i)^2} \frac{\partial^2 w^{(i)}}{\partial x^2} + \frac{\partial^2 w^{(i)}}{\partial y^2} = 0 \tag{6-1}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left[\phi^{(i)} - \frac{e_{15}^{(i)}}{\lambda_{11}^{(i)}} w^{(i)} \right] = 0 \tag{6-2}$$

where

$$\alpha^{(i)} = \sqrt{1 - \frac{v^2}{V_C^{(i)^2}}}, \quad (i = 1, 2) \tag{7}$$