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Mixed-mode fracture & non-planar fatigue analyses of cracked I-beams, using a 3D SGBEM–FEM Alternating Method

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ABSTRACT

In the present paper, computations of mixed mode stress intensity factor (SIF) variations along the crack front, and fatigue-crack-growth simulations, in cracked I-beams, considering different load cases and initial crack configurations, are carried out by employing the three-dimensional SGBEM (Symmetric Galerkin Boundary Element Method)–FEM (Finite Element Method) Alternating Method. For mode-I cracks in the I-beam, the computed SIFs by using the SGBEM–FEM Alternating Method are in very good agreement with available empirical solutions. The predicted fatigue life of cracked I-beams agrees well with experimental observations in the open literature. For mixed-mode cracks in the web or in the flange of the I-beams, no analytical or empirical solutions are available in the literature. Thus mixed-mode SIFs for mixed-mode web and flange cracks are presented, and non-planar fatigue growth simulations are given, as benchmark examples for future studies. Moreover, because very minimal efforts of preprocessing and very small computational burden are needed, the current SGBEM–FEM Alternating Method is very suitable for fracture and fatigue analyses of 3D structures such as I-beams.

1. Introduction

The calculation of fracture mechanics parameters (such as the Mode I, II and III stress intensity factors), for arbitrary surface and embedded cracks in complex 3D structures, remains an important task for the structural integrity assessment and damage tolerance analyses [1]. The application of linear elastic fracture mechanics to fracture and fatigue analyses of various types of structures has been hindered by the lack of analytical solutions of stress intensity factors, which characterize the magnitude of the singular stress field near the crack-tip/crack-front. The strength of a cracked structure and the fatigue crack growth rate under cyclic loading can be determined once the corresponding stress intensity factors are computed. A rational fatigue life estimation of the cracked structure can be made based on these analyses, for the design, maintenance and damage tolerance of civil, mechanical and aerospace structures.

I-beams are widely used as typical structural components in structural engineering. Since the I-beam has the characteristics of both three-dimensional finite bodies as well as slender bars, it is quite difficult to find the exact solution for stress intensity factors for cracked I-beams by the existing classical analytical methods. By applying the conservation laws and elementary beam theory, Kienzler and Hermann [2] obtained the approximate stress intensity factors for cracked beams with different crack geometries and a rectangular cross section. Hermann and Sosa [3] applied the method in [2] to find the stress intensity factors of cracked pipes under different loading conditions. The stress intensity factors are normally expressed as a function of the finite-size correction factor β in literature [2]. Gao and Hermann [4] applied asymptotic techniques based on the method introduced in [2] to calculate a better approximation for the correction factor β. Müller et al. [5] extended the method of Kienzler and Hermann [2] to assess the stress intensity factors of circumferentially cracked cylinders and rectangular beams under different loading conditions. Dunn et al. [6] calculated the stress intensity factors of I-beams under a bending moment, by extending the work of [2] along with dimensional considerations and a finite element calibration.

Another approach for determining stress intensity factors of engineering structures is by using the G*-integral, based on conservation laws and the concept of crack surface widening energy release rate. Xie et al. [7–9] showed that the crack surface widening energy release rate can be expressed by the G*-integral along with

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elementary strength of materials theory for slender cracked structures. They applied the G*-integral to calculate the stress intensity factors of thin-walled tubes with a cracked rectangular cross section, and they also applied this method to analyze homogeneous and composite multi-channel beams [10]. Kishen and Kumar [11] employed a cracked beam-column element, introduced by Tharp [12], in a finite element simulation to study the fracture behavior of cracked beam-columns with different load eccentricities.

It should be noted that, all the above-mentioned analytical or empirical solutions can only be used for analyzing symmetric model-I cracks in I-beams. They are not valid for fatigue crack growth simulations of mixed-mode cracks in I-beam, such as the inclined cracks in the flanges. Numerical methods are necessary for fracture and fatigue analyses of mixed-mode cracks in complex 3D structures.

In spite of its wide-spread popularity, the traditional Finite Element Method (FEM), with simple polynomial interpolations, is unsuitable for modeling cracks and their propagations. This is partially due to the high-inefficiency of approximating stress & strain-singularities using polynomial FEM shape functions. In order to overcome this difficulty, embedded-singularity elements by Tong et al. [13], Atluri et al. [14], and singular quarter-point elements by Henshell and Shaw [15] and Barsoum [16], were developed in order to capture the crack-tip/crack-front singular field. Many such related developments were summarized in the monograph by Atluri [17] and they are now widely available in many commercial FEM software. However, the need for constant re-meshing makes the automatic fatigue-crack-growth analyses, using traditional FEM, to be very difficult.

In a fundamentally different mathematical way, after the derivation of a complete analytical solution for an embedded elliptical crack in an infinite body whose faces are subjected to arbitrary tractions [19], the first paper on a highly-accurate Finite Element (Schwartz–Neumann) Alternating Method (FEAM) was published by Nishioka and Atluri [18]. The FEAM uses the Schwartz–Neumann alternation between a crude and simple finite element solution for an uncracked structure, and the analytical solution for the crack embedded in an infinite body. Subsequent 2D and 3D variants of the Finite Element Alternating Methods were successfully developed and applied to perform structural integrity and damage tolerance analyses of many practical engineering structures [1]. Recently, the SGBEM (Symmetric Galerkin Boundary Element Method)–FEM (Finite Element Method) Alternating Method, which involves the alternation between the very crude FEM solution of the uncracked structure, and an SGBEM solution for a small region enveloping the arbitrary non-planar 3D crack, was developed for arbitrary three-dimensional non-planar growth of embedded as well as surface cracks by Nikishkov et al. [20], and Han and Atluri [21]. An SGBEM 'super element' was also developed in Dong and Aluri [22–25] for the direct coupling of SGBEM and FEM in fracture and fatigue analysis of complex 2D & 3D solid structures and materials. The motivation for this series of works, by Atluri and many of his collaborators since the 1980s, is to explore the advantageous features of each computational method: model the complicated uncracked structures with simple FEMs, and model the crack-singularities by mathematical methods such as complex variables, special functions, boundary integral equations (BIEs), and by SGBEMs.

In the present paper, by employing the SGBEM–FEM Alternating Method, the stress intensity factors of I-beams with different crack configurations are computed; fatigue-crack-growth process of the I-beams subjected to cyclic loadings are examined. The stress intensity factors of the crack-front are computed during each step of crack increment. Crack growth rates are determined by the Paris Law. The crack paths and number of loading cycles are predicted for damage tolerance evaluation. The validations of SGBEM–FEM Alternating Method are illustrated by the comparison of numerical results with available empirical solutions as well as experimental observations. For mode-I cracks in the I-beam, the computed SIFs by using the SGBEM–FEM Alternating Method are in very good agreement with empirical solutions. And the predicted fatigue life of cracked I-beams agrees well with experimental observations in the open literature. For inclined cracks in the web or in the flange of the I-beams, no analytical or empirical solutions are available in the literature. Thus mixed-mode SIFs of inclined web and flange cracks are presented and non-planar fatigue growth simulations are given, as benchmark examples for future investigators.

2. SGBEM–FEM Alternating Method: theory and formulation

The Symmetric Galerkin Boundary Element Method (SGBEM) has several advantages over traditional and dual BEMs by Rizzo [26], Hong and Chen [27], such as resulting in a symmetrical coefficient matrix of the system of equations, and the avoidance of the need to treat sharp corners specially. Early derivations of SGBEMs involve regularization of hyper-singular integrals, see the work by Frangi and Novati [28]; Bonnet et al. [29]; Li et al. [30]; Frangi et al. [31]. A systematic procedure to develop weakly-singular symmetric Galerkin boundary integral equations was presented by Han and Atluri [32,33]. The derivation of this simple formulation involves only the non-hyper singular integral equations for tractions, based on the original work reported in Okada et al. [34,35]. It was used to analyze cracked 3D solids with surface flaws by Nikishkov et al. [20] and Han and Atluri [21].

For a domain of interest as shown in Fig. 1, with source point $\mathbf{x}$ and target point ξ, 3D weakly-singular symmetric Galerkin BIEs for displacements and tractions are developed by Han and Atluri [32].

The displacement BIE is:

$$\begin{aligned}\frac{1}{2}\int_{\partial\Omega} v_p(\mathbf{x})u_p(\mathbf{x})dS_x &= \int_{\partial\Omega} v_p(\mathbf{x})dS_x\int_{\partial\Omega} t_j(\xi)u_j^{*p}(\mathbf{x},\xi)dS_\xi \\ &+ \int_{\partial\Omega} v_p(\mathbf{x})dS_x\int_{\partial\Omega} D_i(\xi)u_j(\xi)G_{ij}^{*p}(\mathbf{x},\xi)dS_\xi \\ &+ \int_{\partial\Omega} v_p(\mathbf{x})dS_x\int_{\partial\Omega} n_i(\xi)u_j(\xi)\varphi_{ij}^{*p}(\mathbf{x},\xi)dS_\xi \end{aligned} \quad (1)$$

And the corresponding traction BIE is:

$$\begin{aligned}-\frac{1}{2}\int_{\partial\Omega} w_b(\mathbf{x})t_b(\mathbf{x})dS_\mathbf{x} &= \int_{\partial\Omega} D_a(\mathbf{x})w_b(\mathbf{x})dS_\mathbf{x}\int_{\partial\Omega} t_q(\xi)G_{ab}^{*q}(\mathbf{x},\xi)dS_\xi \\ &- \int_{\partial\Omega} w_b(\mathbf{x})dS_\mathbf{x}\int_{\partial\Omega} n_a(\mathbf{x})t_q(\xi)\varphi_{ab}^{*q}(\mathbf{x},\xi)dS_\xi \\ &+ \int_{\partial\Omega} D_a(\mathbf{x})w_b(\mathbf{x})dS_\mathbf{x}\int_{\partial\Omega} D_p(\xi)u_q(\xi)H_{abpq}^{*}(\mathbf{x},\xi)dS_\xi \end{aligned} \quad (2)$$

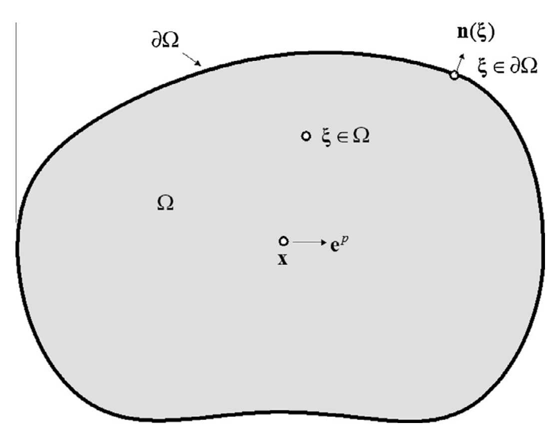

Fig. 1. A solution domain with source point $\mathbf{x}$ and target point ξ.