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A stress-based fatigue criterion to assess high-cycle fatigue under in-phase multiaxial loading conditions

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ABSTRACT

In this paper, a new criterion for in-phase multiaxial fatigue life prediction in high-cycle fatigue is proposed. The new stress-based fatigue criterion is based on the modification of Gough's limit curve. In the case of cyclic plane stress loading, the particular form of the criterion is discussed in detail. The values of parameters in the proposed criterion can be obtained based on fatigue limits in bending and torsion. This allows use of the proposed criterion with almost no limitations to engineering calculations for different metals. Analytical predictions are compared to available experimental data from the literature for six different materials. Additionally, the predictions based on the proposed criterion are compared with the classic Gough's criterion. The predictions based on the proposed criterion seem to be more precise for the analysed materials.

1. Introduction

Despite great progress in fatigue analysis in recent decades, there is a need for reliable methods to predict fatigue life under multiaxial cyclic loading. It is desirable to actually base such a proposal on general principles of solid mechanics and the evaluation of constants contained in the criterion, from the standard uniaxial tests. The multiaxial fatigue tests are complex and expensive. Therefore, several proposals have been made to correlate uniaxial fatigue test data. A review of various failure criteria for multiaxial fatigue is given by Krempl [1], Garud [2], Brown and Miller [3], Macha and Sonsino [4].

These approaches can be divided into three categories: stress-based methods, strain-based methods and energy-based approaches. In the low-cycle fatigue regime, when relatively large plastic strains occur, the strain-based criteria are recommended. In the high-cycle regime, with plastic strain decreasing in the material, the stress-based criteria are often used. To obtain a unified description of both low- and high-regimes, a strain energy strategy has been proposed. This approach is based on the assumption that the energy is proportional to the fatigue damage calculated in different ways. Feltner and Morrow [5] proposed a criterion based on hysteresis energy; Garud [6] presented the plastic strain energy model for multiaxial loading; and Golos and Ellyin [7,8] modelled uniaxial, multiaxial fatigue and cumulative damage in terms of the total strain energy density. Among different energy-based approaches, a non-dimensional parameter that holds terms for material properties was proposed by Varvani-Farhani [9]. This criterion also includes the mean stress effect and additional hardening for out-of-phase loading conditions. The energy-based approach seems to be promising because energy is a scalar quantity and includes both stress and strain components of the loading path.

In the case of stress-based criteria, we can distinguish two main methodologies. The first one is based on the analysis of the cyclic stress invariants, and the second one considers the stress state at the selected plane. The invariant stress criterion can be expressed in a general form as follows [10]:

$$F(I_1, I_2, I_3, N_f) = 0 \quad (1)$$

where $I_1 = \sigma_1 + \sigma_2 + \sigma_3$, $I_2 = \sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3$, and $I_3 = \sigma_1\,\sigma_2\,\sigma_3$; σ_1, σ_2, and σ_3 are principal stresses; and N_f is the number of cycles to failure.

According to the second methodology, fatigue analysis is performed on one critical section in the element. This plane is called critical and is usually different for selected fatigue models. It is often assumed that the failure occurs at a given number of cycles when the function of a normal stress and a shear stress reaches the critical state.

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A brief overview of several stress-based high-cycle fatigue criteria is first discussed in this paper. Several criteria proposed in recent decades are aimed at reducing a given multiaxial stress state to an equivalent uniaxial stress loading. Although there are many proposed criteria for biaxial loading, most of them are limited to specific materials or loading conditions. To the authors' knowledge, there is no existing biaxial fatigue criterion that is universally accepted.

The main goal of this paper is to propose a general form of a criterion for in-phase multiaxial fatigue failure based on the equivalent stress concept. One of the fundamental problems of calculating fatigue strength is the determination of the stress safety space corresponding to a given complex state of stress. Unlike the previous stress-based models, equivalent stress methods for multiaxial in-phase loading, based on the distortion strain energy density, are theoretically correlated with the fatigue limit and depend on the material parameters. This allows the use of the proposed criterion for engineering calculations – with almost no limitations and for different metals. The current criterion is compared with Gough's model using the available experimental data for six different materials and various cyclic load paths.

2. Biaxial stress-based high-cycle fatigue criteria

Because the main focus of this paper is high-cycle fatigue, only the stress-based approach commonly used in high-cycle fatigue design procedures is reviewed in this section. Large numbers of criteria are present in the literature – only a few common criteria are briefly discussed here. Generally, as was mentioned above, the stress-based criteria can be divided into two categories, namely, those based on stress invariants related to the number of cycles to failure (including equivalent stress- and average stress-based models) and those based on critical plane stress (calculated in different ways). Concerning older theories, the authors feel that attention has been taken from the more widely used theories in high-cycle fatigue.

Gough and Pollard [11–13] proposed two criteria for metals under combined in-phase bending and torsion.

For ductile metals, the equation takes the form of an ellipse quadrant:

$$\left(\frac{\sigma}{\sigma_{af,-1}}\right)^2 + \left(\frac{\tau}{\tau_{af,-1}}\right)^2 = 1 \tag{2}$$

where $\sigma_{af,-1}$ and $\tau_{af,-1}$ are fatigue limits in reversed bending and torsion, respectively.

For brittle metals, the ellipse rewritten in the form:

$$\left(\frac{\sigma}{\sigma_{af,-1}}\right)^2\left(\left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)-1\right)+\left(\frac{\sigma}{\sigma_{af,-1}}\right)\left(2-\left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)\right)+\left(\frac{\tau}{\tau_{af,-1}}\right)^2=1 \tag{3}$$

has been proposed.

Findley [14] proposed a criterion based on the linear combination of the axial stress and the shear stress squared (parabola) in the following form:

$$\left(\frac{\sigma}{\sigma_{af,-1}}\right) + \left(\frac{\tau}{\tau_{af,-1}}\right)^2 = 1 \tag{4}$$

Considering the phase difference between loading, Lee [15] modified the ellipse quadrant of Gough as follows:

$$\sigma_{a,eq} = \sigma\left[1 + \left(\frac{\sigma_{af,-1}\ \tau}{\tau_{af,-1}\ \sigma}\right)^2\right]^{1/2} \tag{5}$$

In recent years, criteria based on the critical plane approach have attracted increasing attention [16–19].

Carpinteri et al. [18–21] used the maximum normal stress and the shear stress amplitude on the critical plane as parameters to modify Gough's criterion. The calculation of the critical plane is performed in two steps. First, the weighted mean direction of the maximum principal stress under multiaxial random loading is estimated [20–25]. According to this concept, the fatigue failure assessment is presented by considering a quadratic combination of the maximum normal stress ($\sigma_{\max}$) and the shear stress amplitude (τ_a) in the following form:

$$\left(\frac{\sigma_{\max}}{\sigma_{af,-1}}\right)^2 + \left(\frac{\tau_a}{\tau_{af,-1}}\right)^2 = 1 \tag{6}$$

Several different multiaxial fatigue criteria based on stress invariants are discussed in [26]. The criterion formulated by Sines [27] is probably the most popular invariant-based approach. According to this method, the stress state is in its fatigue limit condition when the following state is achieved:

$$\sqrt{I_{2,a}} + k\sigma_H \le \lambda \tag{7}$$

where I_{2a} is the second invariant of the stress tensor, σ_H is the hydrostatic stress amplitude, and k and λ are material constants that can be calculated by considering two fatigue limits.

Papadopoulos et al. [26] proposed the fatigue model based on the average stress approach, which can be expressed as:

$$\sqrt{\langle T_a\rangle^2} + \lambda[I_{1a} + I_{1m}] = \sqrt{\frac{\sigma_a^2}{3} + \tau_a^2} + \lambda[I_{1a} + I_{1m}] = \zeta \tag{8}$$

where $\langle T_a\rangle^2$ is the average quantity within the volume, I_{1a} is the first invariant of the stress tensor, σ_a is the bending stress amplitude, τ_a is the torsion stress amplitude, and λ and ζ are material parameters depending on the fatigue limit.

Brighenti and Carpinteri [34] proposed a fatigue damage model to evaluate the life assessment of notched structural elements under multiaxial stress histories. Ottosen et al. [35] presented high-cycle fatigue modelling using multiaxial load histories (in-phase and out-of-phase). Stephanov [36] showed a method for multiaxial fatigue life estimation under non-proportional stresses.

3. Equivalent stress based criterion for multiaxial cyclic loading

The stress at the point in the structure subjected to multiaxial cyclic loading can be expressed through the stress tensor as:

$$[\Delta\sigma_{ij}(t)/2] = \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_y(t) & \tau_{yz}(t) \\ \tau_{zx}(t) & \tau_{zy}(t) & \sigma_z(t) \end{bmatrix} \tag{9}$$

This stress state can be expressed in the principal axes of the coordinate system as:

$$[\Delta\sigma_{ij}(t)/2] = \begin{bmatrix} \sigma_1(t) & 0 & 0 \\ 0 & \sigma_2(t) & 0 \\ 0 & 0 & \sigma_3(t) \end{bmatrix} \tag{10}$$

In this paper, the form strain energy density approach presented by Huber has been used to analyse multiaxial fatigue in high-cycle loading. This approach leads to similar conclusions as obtained by von Mises and Hencky. Therefore, here it will be described as Huber–Mises–Hencky's (HMH) hypothesis [28].

According to this approach, failure under cyclic multiaxial fatigue in high-cycle loading is proposed to occur when the maximum distortion strain energy density exceeds the distortion strain energy density at the yield stress obtained from a uniaxial tensile test, i.e., $\phi_f \leqslant \phi_{f,yield}$.