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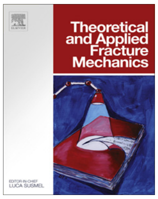

The mixed mode crack growth rate in cruciform specimens subject to biaxial loading

V.N. Shlyannikov *, A.V. Tumanov, A.P. Zakharov

Research Center for Rower Engineering Problems of the Russian Academy of Science, Russia

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ABSTRACT

Cruciform specimens of two configurations with an inclined crack subject to a system of biaxial loads are used to study the fatigue crack growth rate. A method for infiltrating the mixed mode displacement of cracks in the deformed state is suggested. For the particular specimen geometries considered, the *T*-stress and the geometry dependent correction factors, as well as the numerical constant of the plastic stress distributions I_n, are obtained as a function of the dimensionless crack length, load biaxiality and mode mixity. The combined effect of load biaxiality and crack orientation on the crack growth rate for low-strength and high-strength steels is made explicit. Additionally, a comparative study of a cruciform specimen with a working area thinned with respect to a flat cruciform specimen is performed through experiments and numerical computations under various mixed mode biaxial loading conditions.

1. Introduction

According to Brown and Miller [1,2] the fatigue lifetime may be roughly divided into four phases: (i) nucleation of fatigue cracks (defined as initiation); (ii) growth on a plane of maximum shear; (iii) propagation normal to the tensile stress; and (iv) final rupture of the specimen. Although, in the general fatigue problems, considerable attention has been paid to the uniaxial test, there has been comparatively little study into the load biaxiality effects on fatigue crack propagation. The multiaxial fatigue results are of particular importance to practical application, cumulative damage studies and many situations where the principal stress axes can rotate.

There are a number of theories concerning the cause of the phase (ii) to phase (iii) transition which depends on multiaxial loading conditions. Some of these may be disproved by the occurrence of a phase (iii) to phase (ii) transition. Modern multiaxial fatigue failure criteria is based on the critical plane approach which is specified according to the dominating fracture mechanism or crack growth phases mentioned above. Importance of these criteria has increased during last two decades due to of its effectiveness in the assessment of multiaxial fatigue life and application possibilities. The general purpose of critical plane concept is the transformation of a multiaxial stress–strain state to some equivalent iniaxial one and determination of lifetime as well as dominating fatigue crack plane position. In multiaxial fatigue, Carpinteri et al. [3–6] using the critical plane approach have introduced the C–S criterion and highlighted its application to fatigue life estimation and structural integrity assessment of welded joints and components. The authors reported the influence of both proportional (in-phase) and non-proportional (out-of-phase) cyclic loading and proposed the modification of the Carpinteri and Spagnoli (C–S) criterion. These modifications are related to the weighting procedure of the principal stress axes; the definition of the equivalent normal stress by taking into account the mean normal stress effect; the expression of the quadratic combination of stresses. Karolczuk and Macha [7] have published extensive observations of the critical plane positions as a function of multiaxial stress–strain state, combined cyclic or random loading, stress ratio and mean stress. The analysis was concerned to multiaxial failure criteria and the different methods of their formulation (the damage accumulation, the weight function and the variance method).

Systematic studies of both the Shear Stress-Maximum Variance Method and the Theory of Critical Distances (TCDs) have been described by Susmel et al. [8,9]. The orientation of the critical plane is used to estimate fatigue lifetime of both plain and notched engineering components under constant as well as variable amplitude uniaxial/multiaxial fatigue loading. The TCDs is treated as a material property whose length increases as the number of cycles to failure decreases. Shlyannikov [10] considered the behavior of the characteristic distance or damage zone size under both static and cyclic mixed mode fracture on the base of strain energy density theory.

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* Corresponding author.
E-mail address: shlyannikov@mail.ru (V.N. Shlyannikov).

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Nomenclature

a	crack length
E	Young's modulus
G	shear modulus
I_n	numerical constant
J	J-integral
K_1, K_2	mode I and mode II elastic stress intensity factors
K_M^P	plastic stress intensity factor
M_P	mode mixity parameter
n	strain hardening exponent
P_x,P_y	applied to the specimen arm loads in the X-axis and Y-axis direction
R	cyclic stress ratio
r,θ	polar coordinates
S	strain energy density factor
t	specimen thickness
$\overline{T}$	dimensionless non-singular T-stresses
u, v	displacement components
$\bar{u}$, $\bar{v}$	normalized displacement components
$\tilde{u}$, $\tilde{v}$	dimensionless angular functions of displacement components
w	specimen width
Y_1, Y_2	geometry dependent correction factors
α	crack angle
$\bar{\alpha}$	strain hardening coefficient
η	biaxial nominal stress ratio
λ	applied to the specimen arm load ratio
ν	Poisson's ratio
σ_{xx}^{∞}, σ_{yy}^{∞}	nominal stress in the X-axis the Y-axis direction
σ_e	von Mises equivalent stress
σ_0	yield stress

Many different test specimens have been used to investigation of crack growth rate under multiaxial loading on both phase (ii) and phase (iii) crack extention. The cruciform specimen is one of the most used samples for static and cyclic biaxial and mixed mode tests. A review of literature devoted to the application of biaxially loaded specimens in fracture mechanics tests is given by Smith and Pascoe [11]. The cruciform specimen (CS) is very suitable for mixed mode fracture experiments because it reproduces the complete range of mode mixities from pure mode I to pure mode II and has the largest area of uniform nominal stress distributions in working area of specimen. By changing both initial crack angle and the nominal stress biaxiality, different combinations of modes I and II can be achieved.

Miller and Brown [12,13] investigated fatigue crack propagation in biaxially loaded plates. They found that fatigue crack growth is related to two parameters in the plane of plate, the maximum shear stress range and the stress normal to the plane of maximum shear. Both these parameters affected the crack-tip opening displacement and hence fatigue crack growth rate. Shlyannikov et al. [14–16] have published extensive observations of crack growth in compact tension and dual-cruciform specimens. They have shown that the plastic material properties first of all influenced mixed mode crack deviation angle, crack paths and crack growth rate. It should also noted that subject for tests were ten type of aluminum alloys and three type of steels of wide range of elastic–plastic properties. Dalle Donne and Doker [17], testing two cruciform specimen types found that predominant mode II loading drove the stable crack in the direction according to maximum shear strain criterion while subsequent mode I crack-tip loading caused a crack path deviation, that is, the stable crack grew normal to the maximum tensile stress.

A number of studies of fatigue crack propagation have been conducted for biaxial stress conditions with the observations of the T-stress effect. Howard [18] was the first to analyze the crack growth rate by taking into account the T-stress. Kitagawa et al. [19] and Gao et al. [20] have shown that the application of higher stresses and a negative T-stress increases the fatigue crack propagation rates. The generalization of dimensionless $\overline{T}$-stress (normalized by nominal stress $\overline{T} = T/\sigma$) effects for small-scale yield conditions and mixed-mode loading on crack path, fatigue crack growth direction and crack growth rates is given by Shlyannikov [21–23] for materials different properties and specimens of various geometries.

To study the material cyclic fracture resistance characteristics, the mixed mode crack growth rate is calculated as the averaged crack extension per one load cycle versus certain equivalent fracture parameters, namely, the strain energy density (SED) factor. In this paper, the plastic stress intensity factor (SIF) approach, originally proposed to describe fracture toughness for pure mode I under monotonic/static loading, is employed to study the crack growth rate under *cyclic* mixed mode fracture. The elastic mode I and II elastic stress intensity factors K_I and K_2 calculations are supplemented by plastic SIF's determination for a full range of mixed mode conditions, crack length and crack angle combinations. To obtain a plastic stress intensity factor formulation under plane stress and plane strain, the load biaxiality influence is considered and is applied to quantify the crack growth rate in two cruciform specimen types.

2. Specimens and material properties

In the present study, two cruciform specimen (CS) types under biaxial loading were considered. One of them is flat specimen of constant thickness (CS-1, Fig. 1a), the other is a cruciform specimen with a thinned working area (CS-2, Fig. 1b). Different degrees of mode mixity, from pure mode I to pure mode II, are obtained by combinations of the far-field stress level $\sigma = \sigma_{yy}^{\infty}$ in the Y-axis direction, remote biaxial stress ratio η and inclination crack angle α. In the current notation, the magnitude of the applied load biaxiality is described by the remote nominal stress ratio $\eta = \sigma_{xx}^{\infty}/\sigma_{yy}^{\infty}$. By changing α, different combinations of modes I and II can be achieved. For example, it is clear that for the biaxially loaded CS, $\alpha = 0°$ or $\alpha = 90°$ corresponds to pure mode I, whereas pure mode II can be obtained when $\alpha = 45°$ and $\eta = -1$.

The flat cruciform specimens of constant thickness (CS-1) were machined from 3 mm-thick sheets of a low strength structural steel 3, and the cruciform specimens with thinned working area (CS-2) were manufactured from 20 mm-thick plates of fine-grained high-strength steel 34XH3MA. The thickness of the working area was also 3 mm. The most important properties of the tested materials are given in Table 1.

The linear elastic FEM calculations were performed using FE-meshes of both specimen configurations considered (Fig. 2) to determine nominal stress σ_{xx}^{∞}, σ_{yy}^{∞} distributions and a uniform strain field size as well as to determine the relationship between the applied arm load ratio λ and the nominal stress biaxial ratio η under different loading conditions. The commercial finite element code, ANSYS [24], has been used to calculate the displacement and stress distributions ahead of crack tips. 2D plane stress