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Strain energy density prediction of crack propagation for 2D linear elastic materials

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ABSTRACT

When the loading or the geometry of a structure is not symmetrical about the axis of the crack, the rupture occurs in mixed mode loading, and the crack does not propagate in a straight line. It is then necessary to use kinking criteria to determine the new direction of crack propagation.

The aim of this work is to present a numerical modeling of crack propagation under mixed mode loading conditions. This work is based on the implementation of the displacement extrapolation method (DEM) and the strain energy density theory in a finite element code. At each crack increment length, the kinking angle is evaluated as a function of stress intensity factors (SIFs). In this paper, we analyzed the mechanical behavior of inclined cracks by evaluating the stress intensity factors. Then, we present the examples of crack propagation in structures containing inclusions and cavities.

1. Introduction

In fracture mechanics, several numerical approaches were adopted to analyze the mechanical behavior of cracked structures and determine the various parameters of rupture. The finite element method is one of the most numerical methods used today to deal with practical problems of fracture based on the numerical simulation of crack propagation. However, two major difficulties arise with this method:

First, the physical nature of the singular displacement field requires a very fine mesh around the crack-tip in order to properly represent it. The representation of this singularity can be improved by the use the singular elements proposed by Barsoum [1]. These elements provided with additional nodes to a quarter of the sides can integrate exactly the singularity and thus achieve better results.

The representation of the crack poses the problem of crack propagation path by numerical simulation. In the case of cracks under mode I loading, the crack path is fully known as straight. The discretization of the path is then entirely dependent on the mesh size used along it. The technique is commonly known as nodal release technique [2–6]. In the case of a crack under mixed mode loading where crack growth is unknown, two methods are used: remeshing methods [7–11] and methods using interface elements with cohesive zone [12,13].

Various fracture criteria for cracks subjected to mixed mode loading have been introduced for the determination of the propagation direction and the critical stress. Griffith [14] introduced a criterion to determine the conditions to initiate the propagation of a crack. The Maximum Energy Release Rate criterion (MERR) [15,16] followed the Griffith condition and stated that crack growth follows the orientation of maximum energy release rate. Erdogan and Sih [17] developed the Maximum Tangential Stress criterion (MTS-criterion) which was one of the first conditions that predicted critical stress and crack growth orientation. MTS-criterion stated that the crack growth would occur in the direction of the maximum tangential stress and would take place when the maximum tangential stress reaches a critical value which only contains the first mode strength toughness. Due to its simply formulation, MTS-criterion became one of the most popular criteria in fracture mechanics. Palaniswamy and Knauss [18] introduced the G-criterion which dealt with a criterion of maximum energy release rate to determine both the initial crack propagation direction as well as the conditions of crack instability in terms of fracture stress, crack orientation angle, and crack length. Sih [19] proposed the minimum elastic strain energy density factor criterion (SEDF-criterion) which was based directly on the total strain energy density, that is, the sum of its distortional and dilation components.

The strain energy density approach has been found as a powerful tool to assess the static and fatigue behavior of notched and unnotched components in structural engineering [20]. Different SED-based approaches were formulated by many researchers. Labeas et al. [21] presented a methodology to predict mixed-mode

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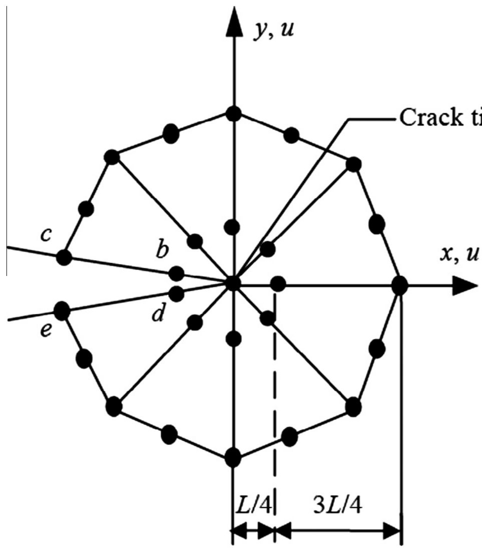

Fig. 1. Special elements used for displacement extrapolation method.

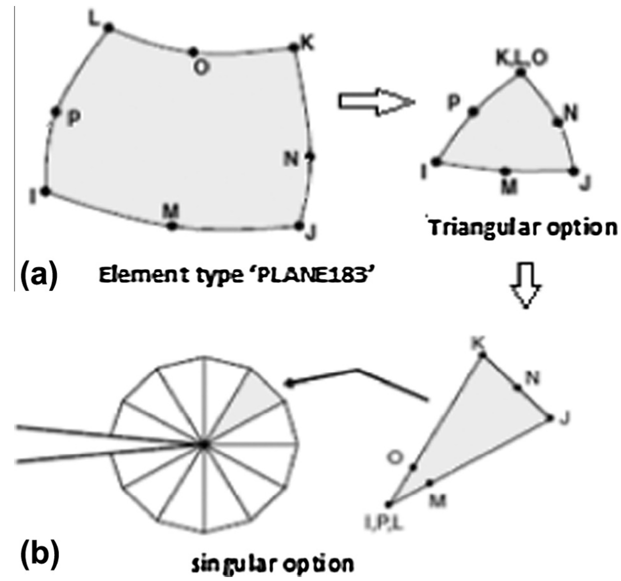

Fig. 3. (a) 'PLANE183' eight-node finite element and (b) singular option.

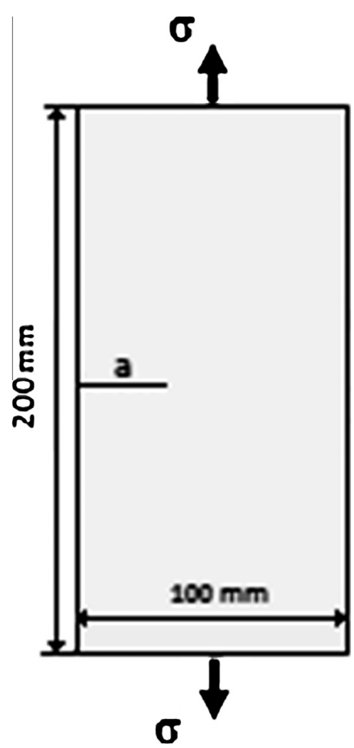

Fig. 2. The geometry of the single edge cracked plate.

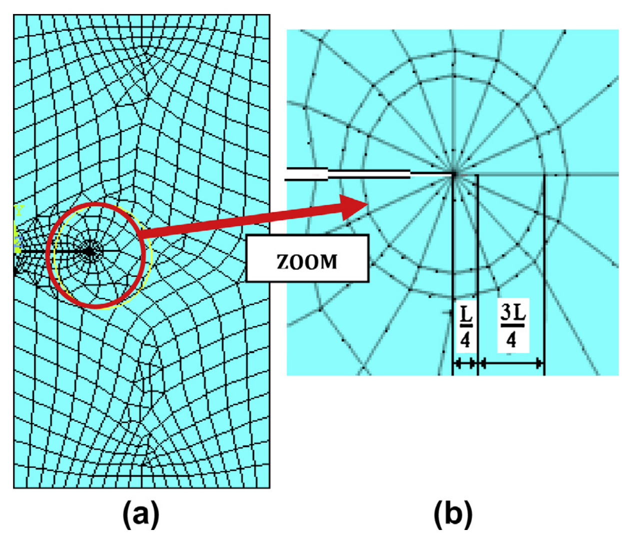

Fig. 4. (a) FE model of the cracked plate and (b) special elements used for displacement extrapolation method.

crack growth and orientation using the strain energy density theory. Zuo et al. [22] developed a fatigue crack growth model to predict the lifetime of fatigue crack growth for single or mixed mode cracks. In Ref. [23], the S-theory is applied to determine crack initiation and direction for cracked T-beams and circumferentially cracked pipes under combined loading conditions. The strain energy density factor approach was also used by Balasubramanian and Guha [24] to analyze the effect of weld size of fatigue crack growth behavior of shielded metal arc welded cruciform joints failing from root region. Ayatollahi and Sedighiani [25] studied the effect of T-stress on the critical mode I stress intensity factor of brittle and quasi-brittle materials. The minimum strain energy density criterion was revisited to take the effect of T-stress into account. In Ref. [26] the strain energy density criterion was applied to determine the direction of crack initiation for various biaxial load factors and material combinations. The scope of this paper is the use of the displacement extrapolation technique and the strain energy density theory to study of crack propagation for various applications, under mixed mode loading conditions.

2. Strain energy density theory and SIFs

Sih [27] has postulated the critical value of the local strain energy as a criterion of crack instability. The minimum of strain energy density around the crack-tip determines the direction of crack propagation. The angle of crack propagation θ can be determined by solving the following equations [28]:

$$\frac{\partial S}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial^2 S}{\partial \theta^2} \geqslant 0 \tag{1}$$