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Multiple cracks subjected to guided waves

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ABSTRACT

The interaction of time harmonic antiplane shear waves and multiple cracks embedded in a plate is studied by using the dislocation model and images method. The effect of the wave number, incident angle and relative positions of the cracks and free surface are presented. Resonance vibrations and the relaxation phenomenon of the layer between the cracks and the free surface are discussed in detail. With the approach, the strain energy density factors (SEDFs) of edge cracks can also be derived by assuming one of the crack tips to be nearly in contact with the free surface.

1. Introduction

The problem of the scattering of elastic waves by edge cracks or near surface defects is of absorbing interest for both nondestructive testing and seismological applications. Considerable progress has been made toward the understanding of the scattering field around damage area. An approximation method was investigated by Stone et al. [1] for the diffraction of a SH-wave by an edge crack normal to the free surface of an elastic half-space. The problem of a subsurface crack subjected to normal and tangential time-harmonic surface tractions was solved by Achenbach and Brind [2]. By determining displacement potentials that satisfy reduced wave equation and specified boundary conditions, the resonance effects for a crack near a free surface were investigated by Keer et al. [3]. The formulation of the problem leads to a system of coupled integral equations which have been solved numerically for the cases of time-harmonic uniform tension and uniform shear. Based on the same technique, dynamic SIFs for an inclined subsurface crack have been computed by Lin et al. [4]. The SIFs for a buried planar and nonplanar cracks in a semi-infinite half-space subjected longitudinal, shear and Rayleigh waves were obtained by Shah et al. [5]. By using boundary integral equations, the scattering of body waves by an inclined edge crack in a half-space has also been investigated by Zhang and Achenbach [6]. The dynamic stress field scattered by arbitrary shaped imperfections at the free surface of an isotropic half-space was analyzed by the method of matched asymptotic expansions developed by Abrahams and Wickham [7]. Dynamic stresses around three parallel cracks in an infinite elastic plate subjected to incident time-harmonic stress waves normal to the cracks have been solved by using the Fourier transform technique by Itou [8].

For most of the analyses mentioned above, the wave scattering by discontinuities were devoted to either half-space or full-space. However, the scattering of waves in a plate-like structure is far more complicated because of multiple reflections. For assuring in-service safety of plate-like structures, the multiple subsurface and edge cracks developed in the structures were often detected by an ultrasonic non-destructive technique. Recently, ultrasonic guided wave inspection is an active technique whereby an ultrasonic pulse is sent to interrogate a long path through the structure. Due to the guiding effect of the plate surface, waves can propagate over distance. When the guided wave hits discontinuities, a typical scattered displacement field is obtained. A change in the scattered field indicates the development of a crack, and thus the growth of such cracks can be monitored. The advantages of using guided waves to examine structural components have shown promises in identifying many types of flaws. For example, defects in pipe support regions where pinhole-type pitting due to corrosion is tended to develop are investigated by a non-dispersive ultrasonic guided wave by Satyarnarayan et al. [9]. For more various analytical approaches and applications, one may refer to literatures [10–15].

Unlike the ultrasonic waves used in conventional ultrasonic inspections that propagate with a constant velocity, the velocity of the guided waves varies significantly with the wave numbers and geometry of the medium, thus the interaction of such waves with damages is complex. Solution of the problem poses difficulties and sometimes it is impossible to obtain in analytical treatment by the presence of the free surface in addition to the crack surfaces and the associated sharp tips. This highlights the essential motivation of providing a better understanding of and a deeper

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insight into the nature of phenomena arising from scattering problems.

In recent years, a number of researchers have worked on this topic. The problem of scattering of guided wave can either be solved by analytical approaches or through the numerical schemes. Considered in [16] is the problem of flexural waves scattering at a through crack in an elastic plate. The interaction of time harmonic elastic waves with an edge crack normal to the plate surface and its depth small compared to plate thickness is investigated by the application of an asymptotic theory of diffraction by Kundu and Mal [17]. To investigate SH wave scattered by a planar edge crack in a plate, a hybrid finite element and eigenfunction technique was used by Datta et al. [18]. Used in [19] is a finite difference method to solve the problems of transient scattering of elastic waves by a normal surface-breaking crack in a plate. Studied in [20] is the flexural wave scattering by a through crack in a plate. With the aid of conformal mapping, the exact solution for a propagation semi-infinite crack in a strip was obtained by Fan [21]. Guided wave scattering by multiple subsurface cracks at arbitrary orientations in a plate was studied by using a hybrid finite element method by Karim [22]. A hybrid numerical method combining finite elements and the boundary integral representation was used by Liu and Datta [23] to investigate the transient scattering of ultrasonic waves by a crack in a plate with its plane impacted by a steel ball. The scattering of a time-harmonic antiplane shear wave by two parallel and coplanar Griffith cracks embedded in an infinite elastic medium is solved by Itou [24] by utilizing a Fourier transformations method. Proposed in [25] is a finite element technique to examine the propagation of guided waves in a flat plate. Recently, guided Lame waves were used [26] for the detection of cracks at rivet hole. The measured scattering filed due to the defects was compared to calculated results obtained by using finite difference method. The work in [27] has conducted numerical approaches based on transient finite element analyses for plates and pipes. The main advantage of these approaches is that complex-shaped waveguides or damages can be handled with standard finite element packages. However, this model is time consuming, which often limits its practical use to practical problems and short propagation distances. The mathematical theory and technology needed to understand the interaction of waves with obstacles, known as multiple scattering, are reviewed by Martin [28]. A variety of techniques, for example separation of variables, and integral equation methods are introduced for solving both the single-obstacle and the multiple-obstacle cases.

In dealing with the scattering field around damage area, especially when multiple cracks are in existence, the dislocation model is one of the powerful tools for the solution [29]. The effects of SH, P and SV waves on the SIFs of two cracks at arbitrary positions in an infinite space and half-space had been analyzed by using a dislocation model developed by Huang [30,31]. In the present study, the dislocation model is extended to determine the problem of wave scattering by multiple cracks in a plate. Numerical results are presented for SIF and SED for various values of wave numbers, wave mode, plate thickness, crack positions and orientations.

There are several proposed methods to predict the direction of crack growth. The most widely accepted methods are the maximum principal stress theory, the maximum energy release rate theory, and the minimum strain energy density (SED) theory [32]. Based on the hypotheses of the SED theory apply to any body in three dimensions, the initial crack growth occurs in the direction along which the strain energy density factor possesses a minimum value $S_{\min}$. The crack extends when the strain energy density $S_{\min}$ reaches a critical value S_{cr}, which can be related direct to the fracture toughness. The concept strain energy density has a wide application in fracture mechanics. Recently, a multiscale fatigue crack growth model [33] was developed to determine the increment of a stress intensity or energy density parameter such that the data in regions I, II and III can be related. By using the available fatigue crack growth data of aluminum panels, the sigmoidally-curved regions I, II and III become a straight line; and the slopes of lines obtained a wider scatter when the energy density range was replaced by the stress intensity range. In our study, due to ease of implementation and demonstrated accuracy, the minimum strain energy density theory will be used to analyze the cases for edge cracks.

2. Formulation of the problem

Consider time harmonic SH-waves which are propagating in an infinite plate of finite thickness $2H$ as shown in Fig. 1.

Two systems of harmonic waves, propagate in a plate by being reflected back and forth between the two free surfaces, can be considered conceivably [34]. One system can be considered as an "incident" wave on the free surface $y = H$, and a "reflected" wave from the surface $y = -H$. The second system can be regarded as a reflected wave from the surface $y = H$, and an incident wave on the surface $y = -H$. If the angle of incidence or the frequency of wave is adjusted properly, the incident and reflected wave within the plate will constructively interfere. In a steady-state situation, the combined wave represents wave motions behaving as standing waves in the y direction and progressive waves in the x-direction. Since these waves penetrate the entire thickness of the plate, a large portion of the medium can be interrogated from a single transducer location. That is the reason why the guided wave inspection is becoming a new emerging technology for rapidly and globally detecting a large area of a structure from a single test location.

In this paper, the function of guided wave as a non-destructive inspection tool will not be discussed. Instead, the effects of the wave frequency and locations of cracks on the scattered stress field will be presented. Since there will be resonance vibrations of the layer between the cracks and the free surface which may substantially give rise to high elevation of local stresses, the SIFs of the cracks will be calculated.

Consider a rectangular Cartesian coordinate system locates at the middle of a plate. Two nonplanar cracks not in contact with each other are located at an arbitrary position in the plate as shown in Fig. 1. All lengths shown in this figure are normalized with respect to the half width of the crack length of crack No. 1. The half crack length of crack No. 2 is denoted by a, $a \leqq 1$. The origins of

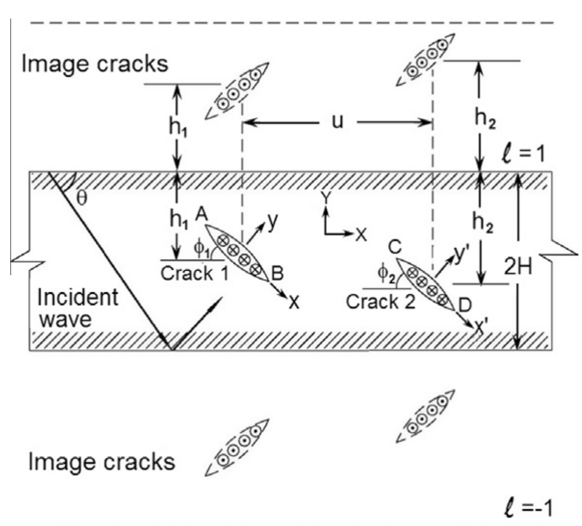

Fig. 1. Geometries of two arrays of vibrating screw dislocations and their images.