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An optimised logarithmic discretisation approach for accurate and efficient compact thermal models

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ABSTRACT

The accuracy of building energy simulations is of considerable interest as discrepant results can elicit adverse financial and environment consequences. The physical and temporal scales considered within building energy applications necessitate compact modelling approaches. The prediction accuracy of such simulations is intrinsically linked with the ability to predict the thermal responses of structural elements. The optimal means of representing these components such that accurate solutions are ensured at minimal computational cost remains unclear. The current study seeks to optimise the spatial placement of nodes through assessing and reporting results pertaining to a logarithmic spatial discretisation method. Contour plots are presented to intuitively determine optimal discretisation levels and time steps required for accurate thermal response predictions. This is achieved by comparing numerical solutions of varying discretisation levels with benchmark analytical solutions. Results are reported in terms of governing dimensionless parameters, Biot and Fourier numbers, to ensure generality of findings. Furthermore, spatial and temporal discretisation errors are separated and assessed independently. Finally, models derived using the proposed guidance achieve high levels of prediction accuracy for typically encountered boundary conditions with buildings.

1. Introduction

Buildings account for approximately 40% of the primary energy consumption within the EU [1,2]. A myriad of options are available to reduce energy consumption in buildings, however, choosing one solution that provides optimal financial or environmental outcomes is a nontrivial matter. This is due to each building having unique operating conditions which are heavily dependent on their design parameters, i.e. climatic conditions, occupancy schedules, architectural designs and the multitude of materials used during construction. The complex nature of building thermal dynamics has motivated the use of simulations as decision-making tools. It is essential that simulations are highly accurate to ensure their effectiveness in evaluating any potential energy saving measures.

Many building energy softwares have been developed and a full review of many of these programs compared and contrasted by Crawley et al. [3]. Probably the most commonly used building energy software packages is EnergyPlus. Developed by the U.S. Department of Energy, EnergyPlus provides a flexible modelling tool capable of simultaneously simulating thermal zones, transient storage effects and energy system demands. The main concepts of EnergyPlus are discussed by Crawley et al. [4]. Mathematical approaches based on the discretisation of governing heat transfer equations are employed, with two solution methods available for simulating thermal responses in structural elements. These are the conduction transfer function (*CTF*) method and the implicit finite difference approach [5]. The CTF method is the default solution approach and employs a space-state solution to obtain heat fluxes at the inner and outer surfaces of structures. The implementation of this approach is based on the work of Seem [6]. Alternatively, the finite difference approach allows for both internal and surface temperatures to be calculated. These two approaches employ the same finite difference discretisation scheme. Studies have shown that the default discretisation approach currently used within the *CTF* approach has certain limitations when applied to situation regarding high thermal mass or low conductivity [7] and when short time steps are used [8].

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Nomenclature

Dimensionless numbers

θ^*	Dimenisonless temperature, $\frac{T_{(t)}-T_{int}}{T_\infty-T_{int}}$
Bi	Biot number, $\frac{hL_c}{k}$
Fo	Fourier number, $\frac{\alpha t}{L_c^2}$
Q^*	Dimenisonless energy, $\frac{Q_{(t)}-Q_{int}}{Q_\infty-Q_{int}}$
x^*	Dimenisonless distance, x/L
$\dot{q}$	Heat flux W

English symbols

$\mathscr{R}$	Overall discretisation effect
$\mathscr{R}_s$	Spatial discretisation effect
$\mathscr{R}_t$	Temporal discretisation effect
$\overrightarrow{F}$	Future temperatures
$\overrightarrow{O}$	All boundary conditions
$\overrightarrow{P}$	Present temperatures
$\overrightarrow{v}$	Eigenvector
A	Area m^2
C_i	i^{th} Coefficient
c_p	Specific heat capacity $J/kg\cdot K$
C_{th}	Thermal capacitance J/K
h	Convective heat transfer coefficient $W/m^2\cdot K$
$H(s)$	Transfer function evaluated for the frequency s
I	Identity matrix
i	Index of element under consideration
j	Index of adjacent element
k	Thermal conductivity $W/m\cdot K$
L	Distance to core of a wall m
L_c	Characteristic length, $L_c = L \pm dx_o$ m
M_o	Coefficient matrix of the vector $\overrightarrow{O}$
n	Number of spatially discrete nodes
Q	Change in internal energy storage J
q''	Volumetric heat generation W/m^3
R_{th}	Thermal resistance K/W
S_A	Coefficient matrix of state space system
T	Temperature K
t	Time s
U	U-value $W/m^2\cdot K$
V	Volume m^3
x	Spatial coordinate m
z	Eigenvalue/eigenvector solution coefficient matrix
a	Distribution exponent
b	Distribution base
dx_o	Displacement of point of inflection from geometric centre m
p	Location of partition between elements

Greek symbols

α	Thermal diffusivity m^2/s
λ	Eigenvalue
ρ	Density kg/m^3

Mathematical symbols

Δ	Discrete distance
∂	Partial derivative
$\rightarrow$	Vector

Subscripts

∞	Ambient
an	Analytical solution
dir	direct solution
int	Initial condition
num	Implicit numerical solution

In a previous study by the authors [9], guidelines were presented on the optimal number of evenly-spaced discretisation nodes required to accurately predict thermal responses in thermal storage media. The results were presented in terms of governing dimensionless parameters, the Biot and Fourier number. Notably, the study showed the importance of considering the boundary conditions, characterised by the Biot number, when determining discretisation levels. While the guidance ensured accurate simulation predictions for local temperatures and energy storage, computational efficiency could be enhanced by optimising nodal positions by utilising uneven nodal spacing.

Optimal computational efficiency is achieved by including the minimal number of nodes required to accurately predict thermal responses. A number of studies have implemented optimisation routines to extend the applicability of models with a pre-defined number of elements. By fixing the number of nodes, attractive computational times can be ensured. The optimisation routines are employed to determine effective thermal resistance and capacitances so that model predictions match either experimental data or benchmark solutions. Both studies by Gouda et al. [10] and Underwood [11] focused on optimising two-node $(3R2C)$ models of typical building constructions. In these studies the values of each thermal resistance and capacitance term was altered to minimise the difference between the predicted surface temperatures from the two-node model and the predictions provided from a detailed reference model. In the study by Underwood, predicted temperatures from a building model derived from his parametric results were compared to those provided by a reference model. The results showed root-mean square errors ranging between $0.1 - 0.5K$ for all surface and zone temperature predictions.

Furthermore, Fraisse et al. [12] examined the use of a four-element model $(3R4C)$ to improve the initial response of simulations to changes in boundary conditions. The two outer elements of this four-element model were each attributed 5% of the total thermal capacitance. In addition to the fourth-order model, two other compact models were assessed. These were: (i) a two-element model with elements placed at the outer surfaces; (ii) and a two-element model with nodes placed in the centre of the cells i.e. the thermal resistance between the outer nodes and the ambient environment include a conductive resistance term. Model parameters were determined analytically and simulation accuracy was assessed through comparison to a one-hundred element benchmark solution. The results showed that the fourth-element model initially achieved higher accuracy compared to the cell-centred two-element model, however, as time progressed both offered near identical results. Both of these cell-centred models surpassed the prediction accuracy of the surface-node model.

Developing discretisation schemes that can be algorithmically applied to all structures has also been explored. These schemes have a fixed method of distributing elements and use discretisation