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Fractal strength of cement gels and universal dimension of fracture surfaces

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ABSTRACT

Compressive strength of porous materials, especially of cement gels, has been estimated by means of fractal dimension of fracture surfaces. The relationship between mechanical strength and fractal characteristics of porous gels has been derived and tested experimentally using samples of cement gels. The dimensions of fracture surfaces have been found to be general parameters independent on the fractal dimensions of inner material components.

1. Introduction

Fracture mechanics and fracture phenomena [1] are subjects of extensive research in the field of material science and physics of stochastic processes. Fracture surfaces of brittle and quasi-brittle materials are characterized by complex morphology that is a consequence not only of the fracture process itself but also the inner material texture and microstructure arrangement. By introducing fractal geometry in fracture phenomena [2] a permanent interest in the fractal analysis of fracture surfaces has been initiated. The fractal method proved to be a fruitful tool for studying morphology and physical properties of fracture surfaces of solids, especially those of metallic nature. Nevertheless, when some authors tried to correlate the fractal dimensions of fracture surfaces of non-metallic porous composites with mechanical strength, they did not largely found convincing results. The reason consists in a more complicated morphology of fracture surfaces of composite materials and this requires a certain refinement of the fractal method, especially as to the statistical and selective approach to the problem. As will be shown in the present paper, porous composite materials may manifest a spectrum of fractal dimensions on their fracture surfaces and this must be carefully accounted for in the analysis. In recent years, it is the porous cementitious materials whose fracture surfaces are subjected to detailed studies and analyses [3].

It is well known that porosity of materials strongly influences their mechanical strength, especially, compressive strength. Pores are inherent part of inner structure. They are formed as a spatial complement to the solid material components that are usually built up of microscopic particles, globules or grains. Porous media of this type represent a broad class of materials widely used in practice. In the past, the pores were mostly investigated independently on their solid environment. They were described as fractals [4] with scaling relations involving their diameters, volumes, number distributions, surface corrugations, etc. However, all those parameters result from the solid environment and this fact should be taken into account in any model describing pore properties.

2. Porosity

Consider a porous material formed by grains of microscopic size. The grains are arranged by fractals in space. Their number distribution N scales are related to their size l as follows:

$$N(l) = \left(\frac{L}{l}\right)^D, \quad l < L, \tag{1}$$

where D is fractal dimension and L correlation length of the fractal arrangement. The grains may form a fractal cluster. The empty space among grains is a pore volume V

$$V \approx L^3 - l^3\left(\frac{L}{l}\right)^D = L^3\left[1 - \left(\frac{l}{L}\right)^{3-D}\right] \approx V_o\left[1 - \left(\frac{l}{L}\right)^{3-D}\right]. \tag{2}$$

The porosity P of the cluster

$$P = \frac{V(l)}{V_o} = 1 - \left(\frac{l}{L}\right)^{3-D}, \tag{3}$$

is only dependent on the fractal dimension D, grain size$l$, and correlation length L of the fractal spatial arrangement. The expression of

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porosity (3) modifies its form if the porous material consists of $(n+1)$ fractal clusters

$$P = \frac{V(l)}{V_o} = 1 - \sum_{i=0}^{n} \left(\frac{l_i}{L_i}\right)^{3-D_i}. \tag{4}$$

However, relation (4) does not take into account the case of a composite material in which the fractal clusters of characteristic sizes L_i can be stochastically scattered and mixed with other phases so that the size Λ of the investigated sample may considerably exceed the cluster sizes $L_i \ll \Lambda$. In order to generalize relation (4), let use suppose that there are m_i fractal clusters with dimension D_i in the sample. Their volume fractions $\xi_i = m_i\, L^3/\Lambda^3$ enables us to calculate the porosity of the whole sample as follows:

$$P \approx \frac{\Lambda^3 - \sum_{i=0}^{n} m_i l_i^3 \left(\frac{L_i}{l_i}\right)^{D_i}}{\Lambda^3} = 1 - \sum_{i=0}^{n} \xi_i \left(\frac{l_i}{L_i}\right)^{3-D_i}. \tag{5}$$

Eq. (5) includes all possibilities of fractal, non-fractal ($D = 3$) or mixed arrangements of the solid environment surrounding the pores. Provided there are $n+1$ components delocalized over the whole sample ($L_i = \Lambda$, $m_i = 1$, $\xi_i = 1$), then Eq. (5) converts back to (4).

3. Compressive strength of porous materials

Compressive strength σ of porous materials, especially those of ceramics, was investigated many times with the conclusion that the mechanical compressive strength is intimately connected with total porosity P as a prime factor besides some others of less importance. These studies resulted in a variety of relations σ (P) the majority of them are equivalent or – at least – similar. For instance, one of the frequently used relation [5] assumes the form of the power function $\sigma = \sigma_o^*(1-P)^k$ and is related to porous metallic ceramic materials, another from [6] is represented by the exponential function $\sigma = \sigma_o^* \exp(-bP)$ which is, in fact, an asymptotic form of the former power formula for $P \to 0$. Other authors [7] suggested the expression $\sigma = \sigma_o^*[1-(P/P_{cr})^{1/a}]$ also similar to that of power form but corrected for a critical porosity P_{cr} at which compressive strength approaches zero. There is also another known expression [8] accounting for the critical porosity $\sigma = \sigma_o^*[1-(P/P_{cr})^e]\cdot[1+f\cdot(P/P_{cr})^g]$. Besides the mentioned relations there are some others [9] but that of the initially mentioned ($\sigma = \sigma_o^*(1-P)^k$) seems to be the basic one among all recommended. Therefore, it would be reasonable to accept it as a background for a further improvement.

Undoubtedly, the concept of critical porosity is an urgent requirement that must be considered. In contrast to some authors' [7] who considered the critical porosity as an absolute threshold beyond which the compressive strength falls down to zero $\sigma(P_{cr}) = 0$, we treat the critical porosity in a bit different way. The critical porosity will represent the value at which the dependency of strength on porosity ceases – in accord with other works [7] – but the structure does not break down because of certain remaining strength s_o caused by some factors that other authors neglected when deriving their equations. Among factors which were neglected and which contribute to the remaining strength s_o there are the pore liquids, especially those of almost incompressible ones like water that make the structure more firm. When a continuously increasing load of higher rate is applied to a rigid porous material with water inside its pores, the water has no time to flow within the pore network and the virtual incompressibility of such a liquid must be taken into account [10]. Therefore, the supposition $\sigma(P_{cr}) = s_o$ seems to be quite reasonable. The corresponding generalized power function then reads

$$\begin{aligned} &\sigma = \sigma_o^*\left(1-\frac{P}{P_{cr}}\right)^k + s_o = \sigma_o(1-P-b)^k + s_o, \\ &0 \leqslant b = 1-P_{cr} \leqslant 1, \quad 0 \leqslant \sigma_o^* \leqslant \sigma_o = \frac{\sigma_o^*}{P_{cr}^k} \end{aligned} \tag{6}$$

where P is total porosity and σ is compressive strength of the sample with virtually incompressible fluid filling at least a part of its pore space.

Combining (6) and (5) the compressive strength of porous matter appears as a function of fractal structure

$$\sigma = \sigma_o\left[\sum_{i=0}^{n} \xi_i \left(\frac{l_i}{L_i}\right)^{3-D_i} - b\right]^k + s_o. \tag{7}$$

4. Fractal compressive strength

With a mixed structure containing both the fractal and non-fractals regions some of the dimensions D_i belong to the volume (mass) fractals ($0 < D_i < 3$) and some to non-fractal solid phases ($D_i = 3$). Similarly, when analyzing fracture surfaces of such multi-component materials, "plane" representatives (D_i^*) of the volume components (D_i) appear on these surfaces. Those projected surface "patterns" preserve the fractal or non-fractal characters of their volume parent objects but their dimensions D_i^* are smaller than those of original objects. Provided a fracture surface has its own dimension S and its morphology being "typical" rather then "special", the relation between D_i^* and D_i can be expressed [11] as follows:

$$D_i^* = \max\{0, D_i - (3-S)\}, \quad D_i^* \leqslant S < 3, \tag{8}$$

where $3-S$ is the co-dimension of the fracture surface. If the fracture surface were the perfect Euclidean plane ($S = 2$), the expression (8) would lead to the well known equation $D_i^* = D_i - 1$. Using (8), the exponent $3 - D_i$ in Eq. (7) can be replaced by $S - D_i^*$ and the generalized Balshin strength function now reads

$$\sigma = \sigma_o\left[\sum_{i=0}^{n} \xi_i \left(\frac{l_i}{L_i}\right)^{S-D_i^*} - b\right]^k + s_o. \tag{9}$$

This function may contain many parameters which makes its fitting to experimental data very difficult because there might be more than one "reliable" set of parameters σ_o, $\{\xi_i\}_{i=1}^n$, l_i, L_i, S, D_i^*, b, k, s_o. Fortunately, the structure of porous material often contains only one type of grains or – at least – one grain type of fractal arrangement ($i = 1$) dominates over the solid rest ($i = 0$) that is usually of non-fractal character ($D_o = 3 \Rightarrow D_0^* = S$)

$$\begin{aligned} &\sigma = \sigma_o\left[\xi_1\left(\frac{l_1}{L_1}\right)^{S-D_1^*} + (\xi_o - b)\right]^k + s_o = \sigma_o\left[\xi_1 \exp((D_1^* - S)/A) - \gamma\right]^k \\ &\quad + s_o, \\ &A = \frac{1}{\ln(L_1/l_1)}, \quad \gamma = (b - \xi_o). \end{aligned} \tag{10}$$

To our knowledge, the fractal analyses of fracture surfaces published so far have not distinguished between the dimensions D_i^* of the projected objects and the dimension S of the fracture surface itself. Here, for the first time clear differences between these two kinds of dimensions are specified and discussed. Although the fracture surface dimension S as a separate parameter and independent on the inner fractal structure may seem to be rather vague, yet, it has clear interpretation as the fracture surface dimension that would be directly measurable if the sample were fully compact (non-porous, i.e. $b = 0$, $s_o = 0$), and, therefore, non-fractal ($D_i = 3 \Rightarrow D_i^* = S$), which means a perfect Euclidean body. Nevertheless, the surface itself may be a fractal with non-integer S, which should even be expected in majority of cases.