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Adaptive response surface method based on a double weighted regression technique

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ABSTRACT

In structural reliability analysis where the structural response is computed from the finite element method, the response surface method is frequently used. Typically, the response surface is built from polynomials whereof unknown coefficients are estimated from an implicit limit state function numerically defined at fitting points. The locations of these points must be selected in a judicious way to reduce the computational time without deteriorating the quality of the polynomial approximation. To contribute to the development of this method, we propose some improvements. The response surface is successively formed in a cumulative manner. An adaptive construction of the numerical design is proposed. The response surface is fitted by the weighted regression technique, which allows the fitting points to be weighted according to (i) their distance from the true failure surface and (ii) their distance from the estimated design point. This method aims to minimize computational time while producing satisfactory results. The efficiency and the accuracy of the proposed method can be evaluated from examples taken from the literature.

1. Introduction

The finite element method is the most efficient numerical tool currently available for the design of new civil engineering structures and the structural analysis of existing constructions. It allows the mechanical equations to be discretized and the nonlinear behavior of materials and structures to be taken into account. The uncertainties affecting the input data of a model, such as geometry, materials and loading parameters of a structure, are of a random nature. A knowledge of the sensitivity to random variables of a result given by the finite element method is useful for structure sizing. It can be obtained by an approach calculating the probability of failure or reliability.

In practice, probabilistic analysis can be applied efficiently only to physical problems in which the numerical modeling does not require prohibitive computational time. Nevertheless, more enhanced nonlinear models that incur higher computation costs are continually being developed. Developing numerical tools which reduce the computational time and provide satisfactory accuracy remains an important issue.

The present paper responds to the problem above-mentioned and should help to enrich the probabilistic methods. It proposes a new response surface method that is accurate and also efficient in terms of the computational time. The efficiency and accuracy of the proposed method are illustrated through three examples taken from the literature.

2. Short literature review

Let us denote by $\bar{x} = [x_1, \ldots, x_n]^T$ the random vector grouping n random variables of a finite element problem (material property, geometry, loading, etc.). The components of this vector have a joint probability density function $f_{\bar{X}}(\bar{x})$ involving correlation between the variables. For each mode of failure of the structure, a limit state function $G(\bar{x})$ is defined in the space of physical variables. The set of variables for which $G(\bar{x}) > 0$ represents the safety domain. The set of variables for which $G(\bar{x}) \leq 0$ represents the failure domain. The frontier $G(\bar{x}) = 0$ is the limit state surface (the failure surface).

The failure probability P_f is defined as:

$$P_f = \int_{G(\bar{x}) \leq 0} f_{\bar{X}}(\bar{x})\, \mathrm{d}\bar{x}. \quad (1)$$

Calculating this integral is not a straightforward task because the joint probability density function $f_{\bar{X}}(\bar{x})$ is not always available. When available, this function cannot always be integrated analytically. The analytic estimation of this integral is impossible if the failure criterion is implicitly defined. The most frequent case is when $G(\bar{X})$ can be computed only by the finite element method.

In order to calculate P_f, it is possible to resort to Monte Carlo simulations [1,2] or approximate methods based on the reliability index β. In practice, the Hasofer-Lind reliability index, noted β_{HL} is often used [3]. This index is defined in the

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standardized space where the random variables are Gaussian reduced and stochastically independent. The index β_{HL} is defined as the minimum distance from the origin of the standardized space to a point of the limit state surface, the so-called design point P^*. The reliability index β_{HL} can be determined by several minimizations under constraint algorithms such as Rackwitz–Fiessler's algorithm [4], Abdo–Rackwitz's algorithm [5]. Some details of these algorithms can be found in a comprehensive report on the state of the art [6].

Among the methods available for assessing structural reliability, the response surface method (RSM) is a useful tool [7,8]. The basic idea of the RSM is to approximate the implicit limit state function $G(\bar{X})$ by an equivalent polynomial function $\tilde{G}(\bar{X})$. Thanks to this approximation, the reliability assessment becomes faster and much more tractable than with the real complex model. Nevertheless, the accuracy of results obtained by this method depends on the quality of the approximate function which has to faithfully reproduce the real limit state function, at least in the vicinity of the failure surface. The coefficients of the approximate function are fitted by the least squares technique from a minimal number of points where the limit state function $G(\bar{X})$ is effectively computed. The set of these points constitutes the experimental design (ED). The quality of the response surface mainly depends on the choice of the response surface form and the selection of experimental design points. Some guidelines have been developed to help in the selection of these points for particular physical models [9,10]. But they do not appear suitable for numerical discrete models. Thus in previous works, the response surface form and the experimental design points have been selected in various ways by different researchers.

Wong [11] employed a factorial experimental design containing 2^n points. He selected values symmetrically around the mean at a distance of one standard deviation. In this technique, the number of fitting points increases rapidly with the number of random variables n.

In order to reduce the number of fitting points in the case where n is large, Bucher and Bourgund [12] proposed an iterative approach to the response surface for reliability analysis. In their work, a quadratic expression without cross terms, which is defined by $(2n + 1)$ coefficients, is employed as an approximate function. In the first step, the mean point is chosen as the central point of the ED. The fitting points are selected, on both sides, along the axes at a distance of some multiples of the standard deviation of the random variables. From this first response surface, a first design point P^* is determined. Then a new central point is obtained as a linear interpolation between the old central point and P^*. A second response surface is then generated around the new central point. This approach requires $(4n + 3)$ evaluations of the limit state function. The authors proposed one update for the response surface.

Rajashekhar and Ellinwood [13] improved the approach by Bucher and Bourgund by considering several update cycles of the coefficients of the response surface in order to obtain satisfactory accuracy and stability of the results. They added cross terms to the response surface expression. Consequently, they obtained better results in the compared examples but, unfortunately, to the detriment of the numerical cost.

Enevoldsen [14] presented an adaptive response surface algorithm in the standardized space called ARERSA (Adaptive Reliability Estimation Response Surface Algorithm). This algorithm utilizes a second order polynomial response surface obtained from central composite designs. First, the domain in which the most likely failure point is located is determined in a global search. In a second stage, a more precise response surface is fitted in the local domain around the most probable failure point obtained from a local search. Then, both FORM and SORM estimates of the reliability are obtained. A complementary procedure is also implemented as a safeguard algorithm to avoid false solutions and a number of checks are suggested to verify the quality of the estimates obtained.

Devictor [15] proposed an algorithm called RSAED (Response Surface with Adaptive Experimental Design) which utilizes polynomial response surfaces in the standardized space. This algorithm takes into account the influence of the stochastic transformation of experimental designs. A database is used to store mechanical calculations which have been already carried out. This database allows the previous points to be reused in order to reduce the number of mechanical calculations. Warning indicators are added to check the quality of the response surface. RSAED allows the geometry of the response surface to be studied in the vicinity of the design point in order to validate results from FORM and SORM.

Based on the work presented in [11–13], an improved RSM was achieved by Kim and Na [16] using the gradient projected technique to choose the fitting points. Thanks to this technique, the experimental design is located in the vicinity of the true limit state surface. In this approach, a linear response surface is employed which could provide a rough approximation if the limit state function is strongly nonlinear.

Drawing on Kim and Na's idea, Das and Zheng [17] developed a cumulative RSM. A linear response surface is initially formed in order to determine the design point P^* by the first order reliability method. The projected gradient technique is used to provide the fitting points. The linear response surface is then enriched by adding square terms, and the second order reliability method is employed to search for the design point. The fitting points defining the linear response surface are reused to produce the quadratic surface. The complementary points are generated around the point P^* obtained from the linear surface. Cross terms can be added to the response surface if necessary.

Gayton [18] proposed a RSM named CQ2RS (Complete Quadratic Response Surface with ReSampling) which allows the knowledge of the engineer to be taken into account. This method is based on a statistical approach and consists in considering the location of the design point P^* as a random variable whose single value is obtained from each resampling of the experimental design. The statistical resampling technique yields an empirical distribution of the coordinates of each design point from the database of computed experiments. A confidence interval can be affected to the mean value. The length of this interval is taken as a criterion for convergence. The first factorial ED is built as close as possible to the design point considering the engineer's knowledge. A resampling technique is then used by removing one point of the ED for each resampling. If the distribution of the design point thus obtained is not satisfactory, new points located inside the confidence interval are added to the ED until convergence is reached. This method reduces the computational time.

Impollonia and Sofi [19] presented an alternative response surface approach for the finite element analysis of stochastic structures with geometrical nonlinearities. This method uses an ad-hoc ratio of polynomials to express the dependence of the response on the uncertain parameters. Thanks to the effectiveness of the response surface form which is insensitive to the locations of the sampling points, the number of sampling points can be reduced.

Like Das and Zheng, Kaymaz and McMahon [20] used a linear response surface for the first iteration and a quadratic response surface without cross terms for the following iterations. The fitting points are generated from the central point. These points are primarily selected in the region where the design point is the most likely to exist in order to reduce the design size, by utilizing sign evaluation of the limit state function. The coefficients of the response surface are determined by a weighted regression technique. A particular system of weighted values of the limit state