Probabilistic Engineering Mechanics 24 (2009) 179–189

Contents lists available at ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Polynomial approximation of aerodynamic coefficients based on the statistical description of the wind incidence

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ARTICLE INFO

Article history:
Received 6 September 2007
Received in revised form
6 May 2008
Accepted 13 May 2008
Available online 21 May 2008

Keywords:
Aerodynamic coefficient
Non-linear aerodynamics
Wind incidence probability
Quasi-steady model
Stochastic linearization
Bridge aerodynamics

ABSTRACT

In civil engineering applications, the aerodynamic coefficients are usually measured in wind tunnels for several wind incidences. The measurement results need to be linearized in order to perform the design of the structure. This paper justifies the use of different linearization techniques for different assessments as divergence or buffeting analysis. In this latter context, it is proposed to linearize the aerodynamic coefficient by the least-square method, using the probability density function of the wind incidence as a weighting function. First this probability density function is computed for a 2-D wind flow, as a function of the wind intensities and their correlation. Then, the comparison of results from different linearization techniques provides surprising results indicating that what is usually performed should be considered with care.

1. Introduction

Despite recent advances in the numerical simulation of fluid-structure interactions (e.g. [1]), the complete interaction between a turbulent oncoming flow and a bridge deck, spread out along several hundreds of meters, still requires heavy computation efforts. The safety of a bridge deck is therefore usually assessed with respect to a list of phenomena, considered separately. Different wind loading models are used for each assessment [2–4]. Two major families of models are distinguished. The first one provides a precise description of the transient self-excited forces by means of flutter derivatives (Scanlan coefficients). The second family gathers quasi-steady models based on aerodynamic coefficients. These models can eventually allow some kinds of non-linearities and are used for evaluating static divergence, galloping or buffeting analysis.

Because of the complexity of bridge deck sections, both flutter derivatives and aerodynamic coefficients need to be measured in wind tunnels. Emphasis is generally put on the estimation of the former one only. Indeed flutter derivatives, resulting from a complex dynamic wind-structure coupling, require dedicated identification procedures [5,6]. However the aerodynamic coefficients are obtained by a simple measurement of the stationary force applied by the wind flow on the bridge deck, for different values of the wind incidence [7]. The variations of the identified aerodynamic coefficients with respect to the wind incidence are typically non-linear irregular functions. The main scope of this paper is to present how to transform the gross measurement results into a format useable for the design.

At the design stage, the identified Scanlan coefficients are used without any further modification: critical flutter wind velocities are obtained as a direct result of the measured coefficients. On the contrary, the gross results of aerodynamic coefficient measurements are not readily useable for the subsequent assessments. The quasi-steady loading is indeed usually linearized. This requires the aerodynamic coefficients to be linearized too, with respect to the wind incidence. The impact of this linearization was first investigated in the late 1980's [8,9] but was limited to 1-D wind flows. Recently it has been shown, in the context of 2-D and 3-D turbulent flows, that higher order polynomial approximations of significantly non-linear aerodynamic coefficients have to be considered [10]. For these reasons, it is necessary to estimate at least a linear, but also any other polynomial, approximation of aerodynamic coefficients. This paper focuses mainly on providing a linear approximation of the measured coefficients, but the procedure is presented in such a general way that it could be applied to supply any higher order formulation.

The most appropriate linearized form of a measured aerodynamic coefficient may be different from one kind of evaluation

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0266-8920/$ – see front matter
doi:10.1016/j.probengmech.2008.05.002

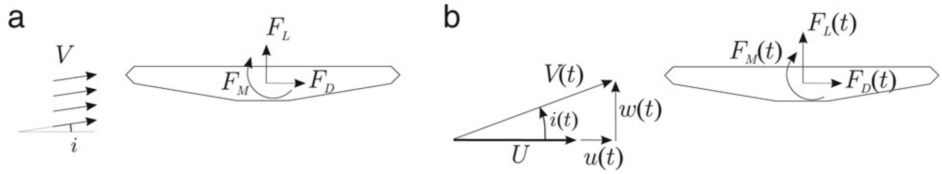

Fig. 1. (a) Aerodynamic forces (drag, lift and moment) on a bridge deck in a uniform laminar flow (b) Extension to a flow with few turbulence, quasi-steady loading.

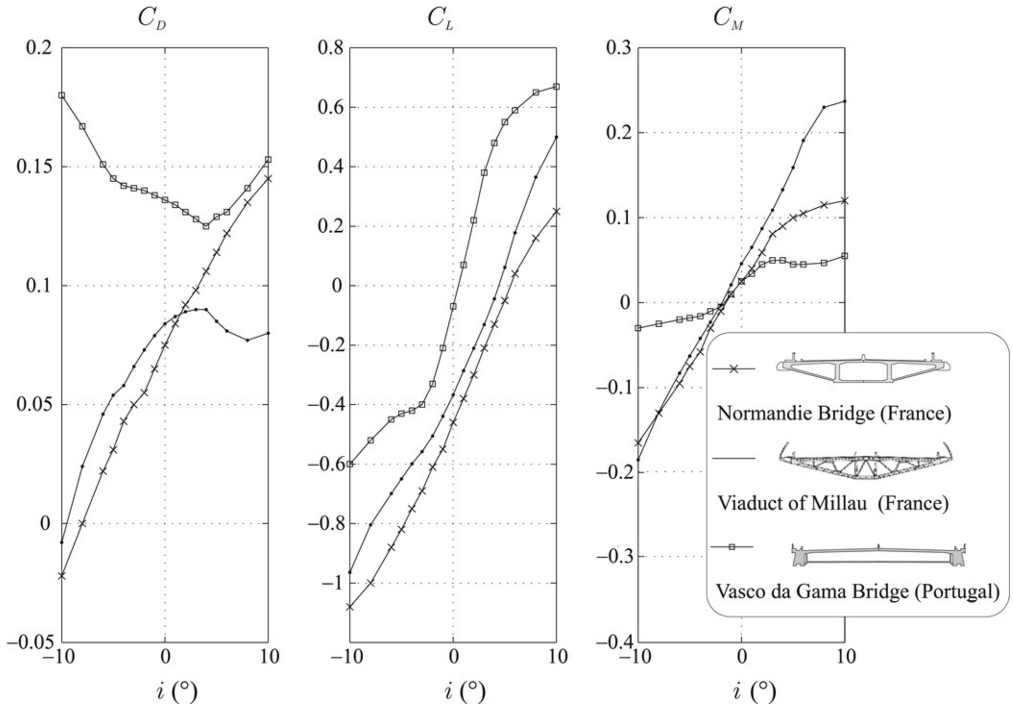

Fig. 2. Examples of aerodynamic coefficients as a function of the wind incidence.

to another. For instance, values around the mean incidence are of major concern when torsional divergence of the bridge deck section (infinitesimal static stability) is considered. In this case a Taylor–McLaurin series expansion around the mean incidence is the best approximation. In the context of a buffeting analysis, the wind incidence changes continuously and it seems logic to weight the measured coefficients by the probability of occurrence of each incidence. A direct application of the stochastic linearization technique [11] results in this conclusion too. It is therefore proposed in this paper to perform the polynomial approximation of the measured aerodynamic coefficients by the least square fitting method with a weighting function proportional to the probability density function (pdf) of the wind incidence.

First, founded on a solid mathematical base, the statistical distribution of the wind incidence is established (2-D wind flow). Since the rigorous expression cannot be used conveniently in practical applications, a simplified version, obtained by a fitting of numerical results, is proposed and its efficiency is demonstrated. The least square fitting of aerodynamic coefficients is then briefly presented in Section 5 with the proposed probability density function used as a weighting function. Finally, the relevance of the proposed technique is demonstrated with a numerical application.

2. Context, motivations

The forces acting on a fixed rigid body immersed in a surrounding flow are expressed as (Fig. 1-a):

$$F = \frac{1}{2}\rho_f C B V^2 \tag{1}$$

where ρ_f is the density of the fluid, C is the aerodynamic coefficient (drag, lift or moment), B is a characteristic width and V is the constant fluid velocity. Due to the variability of considered cross-sections, the aerodynamic coefficients of bridge deck sections have to be measured in wind tunnels for every new project. Their dependence upon the wind incidence i is generally non-linear. Fig. 2 depicts some measured aerodynamic coefficients as a function of the wind incidence.

Civil and structural engineering structures are built in the atmospheric boundary layer, which is known to be turbulent. The wind velocity is not constant in time. In 2-D applications, it is thus composed of a mean velocity U and two zero-mean fluctuations $u(t)$ and $w(t)$ (Fig. 1-b). They are usually modelled as Gaussian processes, with a joint probability density function expressed by:

$$p_{uw}(u, w) = \frac{1}{2\pi U^2 I_u I_w \sqrt{1-\rho^2}} \mathrm{e}^{\frac{-1}{2\left(1-\rho^2\right)U^2}\left(\frac{u^2}{I_u^2} - \frac{2\rho u w}{I_u I_w} + \frac{w^2}{I_w^2}\right)} \tag{2}$$

where $I_u = \frac{\sigma_u}{U}$ and $I_w = \frac{\sigma_w}{U}$ are the turbulence intensities and ρ is the correlation coefficient between both turbulence components.

As a result, the wind incidence $i(t)$ is also a random process. Its instantaneous value changes continuously (Fig. 1-b) but, in a quasi-steady approach, Eq. (1) is still used to express the aerodynamic force applied on the deck. It postulates that the force at a given time depends on the wind components at the same time only, which is valid provided the velocity of the deck is low:

$$F(t) = \frac{1}{2}\rho_f C\left[i(t)\right] B V^2(t)\,. \tag{3}$$