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The effect of fatigue-induced crack propagation on the stochastic dynamics of a nonlinear structure

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Abstract

Reliability design of various structures such as ships, offshore platforms, aircrafts, buildings is incomplete without considering fatigue-induced crack propagation effects that may lead to system failure. In order to carry out dynamic analysis of such systems, one needs an appropriate dynamic model along with a tool for statistical analysis of the crack propagation. This paper aims to contribute to the ongoing research of stochastic dynamics coupled with simultaneous degradation of the system properties, especially stiffness degradation, caused by cracks in structural elements. The latter issue is important for structural safety.

Keywords: Stochastic dynamics; Reliability; Stiffness degradation; Fatigue crack; Path integration

1. Introduction

Dynamic loading of mechanical and structural systems, including randomly varying excitation, causes irreversible changes in the material structure and results in decreasing of the ability of a system to carry intended loading. Such a degradation of system properties has a direct effect on its safety and reliability. Damage caused by vibration, which manifests itself primarily in stiffness degradation of the components in metallic structures, is mainly due to accumulation of fatigue or, more specifically, due to growth of fatigue cracks which takes place in vibrating structural elements.

The analysis of random vibration problems coupled with degradation meets with serious difficulties. Due to this fact it is not easy to construct an analytical and general method for the solution of such problems. Previously, the latter issues were studied in the literature under simplifying assumptions enabling semi-analytical solutions, see e.g. [1,2]. For a general overview of vibration of cracked structures with a 'frozen' damage configuration, cf. [3].

The goal of the present paper is to apply purely numerical techniques, which are general and capable of tackling various system nonlinearities, dynamically evolving degradation along with multi-dimensionality. It appears that numerical path integration [4,5] offers an accurate and efficient solution for three-dimensional random dynamical system with stiffness degradation, which is the topic of the present paper.

2. Stiffness degradation due to fatigue: Governing equations

In this paper we consider a vibrating system, consisting of a thin rectangular plate (cf. Fig. 1), with a center crack orthogonal to the external Gaussian white noise excitation $w(t)$ [1].

The plate itself is considered homogeneously elastic and massless, supporting an ideally stiff heavy mass M at its end. The actual (two-sided) crack size is denoted by $2a$. However, throughout the paper we shall refer to the crack size by using the one-sided crack size parameter a. During vibration the crack grows in the straight initial direction. Let us denote by $k(a)$ the stiffness dependence on the crack size a, which is assumed to have an initial value a_0. The governing equation has the form

$$M\ddot{y}(t) + G_d(y(t), \dot{y}(t))\dot{y}(t) + k(a)y(t) = w(t), \qquad (1)$$

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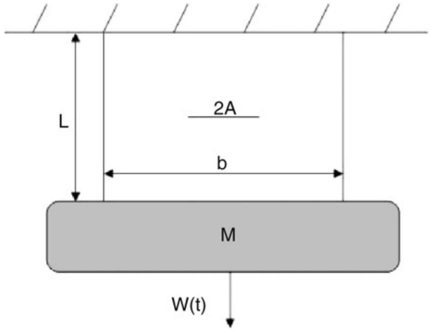

Fig. 1. Schematic illustration of dynamic system.

where $w(t)$ is a random process assumed to be a stationary Gaussian white noise. The response process $y(t)$ characterizes the displacement; M is the mass and G_d is a nonlinear function characterizing the damping in the system. Dividing both sides of Eq. (1) by M and introducing new non-dimensional variables $Y = y/\sigma_y$, $A = a/b$, $\tau = \omega_0 t$ where σ_y denotes the standard deviation of the stationary response of the system (1) without degradation (i.e. when $k(a_0)/M = \omega_0^2$), b is the plate width, we obtain a dimensionless form of Eq. (1)

$$\ddot{Y}(\tau) + G(Y(\tau), \dot{Y}(\tau))\dot{Y}(\tau) + \omega^2(A(\tau))Y(\tau) = W(\tau), \qquad (2)$$

with initial conditions,

$$Y(\tau_0) = Y_0, \quad \dot{Y}(\tau_0) = Y_{1,0}, \quad A(\tau_0) = A_0 = a_0/b, \qquad (3)$$

where $Y(\tau)$ is an unknown response process; $A(\tau)$ is the fatigue crack size; $G(Y(\tau), \dot{Y}(\tau)) = (\omega_0 M)^{-1} G_d(\sigma_y Y(\tau), \omega_0 \sigma_y \dot{Y}(\tau))$ again characterizes damping, $\omega(A)^2 = k(bA)/k(bA_0)$ denotes the monotonically decreasing dependence of the (normalized) stiffness (or undamped natural frequency) on the crack size; $W(\tau)$ is a rescaled white noise. Y_0, $Y_{1,0}$, A_0 are given initial values (or random variables) of the response and degradation. Evolution of the fatigue crack size A is commonly described by the Paris–Erdogan equation

$$\frac{\mathrm{d}A}{\mathrm{d}N} = C(\Delta K)^m \qquad (4)$$

where ΔK is the stress intensity factor range, N is the cycle number, C and m are empirical constants. As is known, the stress intensity factor K can be interpreted as a quantity which characterizes the stress distribution around the crack tip. In general, it can be expressed in the form,

$$K = \beta(A) S \sqrt{\pi A}, \qquad (5)$$

where S describes the far-field stress resulting from the response process Y and $\beta(A)$ accounts for the geometry of the crack and the specimen.

We now define a degradation measure which is similar to the one introduced in [1]. It is based on a nonlinear transformation $\psi(A)$ of A defined as

$$\psi(A) = \int_{A_*}^{A} \frac{\mathrm{d}x}{\omega(x)\beta(x)^m(\sqrt{\pi x})^m}, \qquad (6)$$

where A_* is a suitably defined initial crack size at some time τ_*. For example, A_* could represent a detection level crack size. For the dynamic structure considered, there will also be a critical crack length $A^* \le b/2$, which may be considered as a failure limit state. Let $\psi^* = \psi(A^*)$, and define the degradation measure D as

$$D = \frac{\psi(A)}{\psi^*}, \quad D \in [0, 1]. \qquad (7)$$

Note that for practical reasons the starting time of our analysis is $\tau_0 > \tau_*$, chosen so that $A_0 = A(\tau_0) > A_*$. That is, instead of starting with zero damage, an initial value $D_0 > 0$ will be used.

For the subsequent modelling, a differential equation for the time evolution of D is needed. To derive this equation, the number of cycles $N(\tau)$ in the time interval (τ_0, τ) as a function of τ is required. This relation can be expressed as

$$N(\tau) = \int_{\tau_0}^{\tau} \frac{\omega(A(s))}{2\pi}\mathrm{d}s, \qquad (8)$$

which leads to $\mathrm{d}N(\tau)/\mathrm{d}\tau = \omega(A(\tau))/2\pi$. The differential equation we seek is now obtained as follows,

$$\frac{\mathrm{d}D}{\mathrm{d}\tau} = \frac{1}{\psi^*}\frac{\mathrm{d}\psi(A)}{\mathrm{d}A}\frac{\mathrm{d}A}{\mathrm{d}N}\frac{\mathrm{d}N}{\mathrm{d}\tau} = \frac{C}{2\pi\psi^*}(\Delta S_Y)^m, \qquad (9)$$

where ΔS_Y is the stress range generated by the response process $Y(\tau)$.

It is assumed that the degradation starts when the response process $Y(\tau)$ is in its stationary state and that the damping in Eq. (2) is low. This is expressed by the requirement that $\zeta = \mathrm{E}[G(Y, \dot{Y})]/(2\omega(A)) \ll 1$. This will ensure that the response can be characterized as a narrow-band process in the sense that the envelope process

$$H(\tau) = \sqrt{Y(\tau)^2 + \dot{Y}(\tau)^2/\omega(A)^2}, \qquad (10)$$

serves to quantify accurately the amplitude process associated with $Y(\tau)$.

Denoting the response range within a response cycle by ΔY, that is, $\Delta Y = Y_{\max} - Y_{\min}$, it is assumed that $\Delta S_Y = c\Delta Y$, where the constant c is determined by the dynamical system under consideration. We now make the approximation that ΔY equals two times the amplitude of $Y(\tau)$, as given by the envelope $H(\tau)$ of $Y(\tau)$, i.e. $\Delta Y = 2H$. Therefore the stress range ΔS_Y in the response cycle with amplitude H is given by

$$\Delta S_Y = 2cH. \qquad (11)$$

Eqs. (9) and (11) give the result

$$\frac{\mathrm{d}D}{\mathrm{d}\tau} = C_1 H^m \qquad (12)$$

where C_1 is a suitable constant. $C_1 \sim 1/T_D \ll 1/T$ since the degradation process time scale T_D is much larger than the response period (or the dynamical system time scale)

$$T = \frac{2\pi}{\omega(A)} \qquad (13)$$

Since we shift from A to D by Eq. (7), the stiffness $\omega^2(A)$ in Eq. (2) is replaced by some function $q(D)$. Thus one arrives at the following dynamic equation