Available online at www.sciencedirect.com
ScienceDirect
Theoretical and Applied Fracture Mechanics 46 (2006) 217–231
theoretical and applied fracture mechanics
www.elsevier.com/locate/tafmec

Orthotropic semi-infinite medium with a crack under thermal shock

A.A. Rizk *

Department of Science in Engineering, Faculty of Engineering International Islamic University Malaysia, Kuala Lumpur, Malaysia

Available online 25 October 2006

Abstract

A cracked orthotropic semi-infinite plate under thermal shock is investigated. The thermal stresses are generated due to sudden cooling of the boundary by ramp function temperature change. The superposition technique is used to solve the problem. The crack problem is formulated by applying the thermal stresses obtained from the uncracked plate with opposite sign to be the only external loads on the crack surfaces as the crack surface tractions. The Fourier transform technique is used to solve the problem leading to a singular equation of the Cauchy type. The singular integral equation is solved numerically using the expansion method. The influence of the material orthotropy on the stress intensity factors is shown by comparing the results obtained for different orthotropic materials and isotropic materials in the case of plane stress. The numerical results of the stress intensity factors are demonstrated as a function of time, crack length, location of the crack and the duration of the cooling rate.

Keywords: Orthotropic material; Fracture mechanics; Thermal stresses; Stress intensity factor

1. Introduction

The composite materials have been widely used recently in many engineering applications. The important failure mode in many applications is the cracking of the material under thermal stresses. As many engineering components can be treated as a plate, the study of such cracked plate is quite important. If the crack length is small compared to the width of the plate, the problem can be analyzed as a crack in a semi-infinite plate. Due to surface cooling, high tensile stresses arise near the cooled surface which may cause crack propagation resulting in severe damage. The important correlation parameter in analyzing the subcritical crack growth is the stress intensity factor which can predict whether catastrophic failure will occur due to unstable crack propagation.

Studies of the cracked plate for isotropic elastic plates under transient thermal stresses had been conducted by many researchers [1–7]. For orthotropic materials, the effect of the material orthotropy on the stress

* Tel.: +603 6196 4569; fax: +603 6196 4465.
E-mail address: abdelfattah@iiu.edu.my

0167-8442/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.tafmec.2006.09.001

intensity factors under mechanical loading had been considered in the literature [8–15]. In these studies, the results showed that there are differences between the stress intensity factors obtained from isotropic and orthotropic strips with the same crack geometry and external loads. The analysis of the thermoelastic crack problems in anisotropic materials is also considered in the literature. Determined in [16] are the stress intensity factors for an infinite anisotropic material with a crack caused by an arbitrary distribution of temperature or heat flux over the crack surfaces. A crack in an infinite orthotropic plate under uniform heat flow has been investigated in [17] and a pair of coplanar central cracks in an infinite orthotropic plate under uniform heat flow in [18]. The transient thermal stress problem in an orthotropic thin plate with a Griffith crack perpendicular to the surface of the plate is considered in [19] using the Airy stress function. The solutions of the thermoelasticity problem of an orthotropic plate with two collinear cracks subjected to uniform and linear heat flow on the crack surfaces are investigated in [20]. The thermoelasticity problems of symmetrical and antisymmetrical heat flow disturbed by three coplanar cracks in an infinite orthotropic plate are considered in [21,22]. Thermal stress intensity factors have been obtained [23] for a crack in an infinite orthotropic layer situated parallel to the plane surfaces of the layer for two kinds of boundary conditions with respect to the temperature field. In the first problem, the upper surface of the layer is heated to maintain a constant temperature T_0 while the lower surface of the layer is cooled to $-T_0$. In the second problem, a uniform heat flow is applied to the upper surface of the layer and the same amount of heat is removed from the lower surface of the layer.

In the present work, the analysis of the elastic homogeneous orthotropic semi-infinite plate with an internal and edge crack perpendicular to the boundary under thermal shock is investigated. The thermal stresses are created as a result of sudden cooling on the boundary by a ramp function temperature change. The formulation of the problem is carried out by using the superposition technique. First, the transient thermal stresses for the uncracked medium are obtained under appropriate initial and boundary conditions. Second, the isothermal cracked medium (the perturbation problem) is solved utilizing the thermal stresses obtained from the uncracked plate with opposite sign on the crack surfaces as the crack surface tractions. In this analysis, the problem is assumed to be uncoupled quasi-static, i.e. the inertia effects are neglected and the thermoelastic coupling effects and the temperature dependence of the thermoelastic coefficients are negligible. The Fourier transform technique is used to formulate the perturbation problem leading to a singular integral equation of the Cauchy type with an unknown function which is defined as the derivative of the crack surface displacement. The singular integral equation is solved numerically using the expansion method developed in [24,25]. Numerical results of the stress intensity factors in the case of plane stress for an internal and edge crack are calculated for different orthotropic materials and presented as a function of time, crack geometry, and the duration of cooling rate and compared with the isotropic results.

2. Problem formulation

Consider a homogeneous orthotropic semi-infinite plate at uniform temperature T_∞ with a finite crack of length $l = (b - a)$ perpendicular to the boundary $y = 0$ as shown in Fig. 1a. It is assumed that the principal

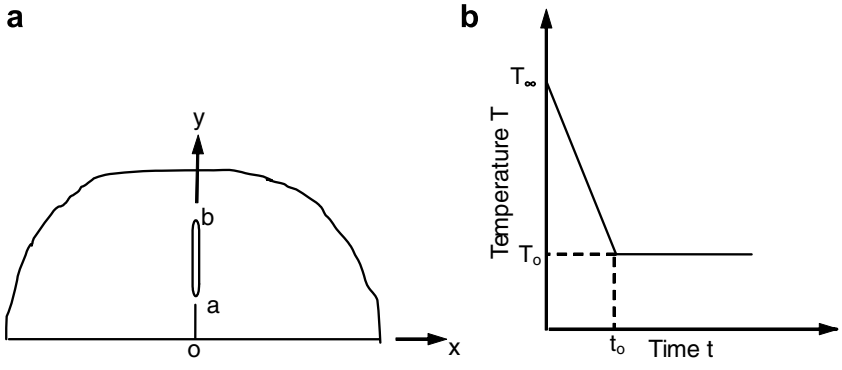

Fig. 1. Crack geometry and temperature boundary condition.