Available online at www.sciencedirect.com
ScienceDirect
Probabilistic Engineering Mechanics 22 (2007) 206–218
PROBABILISTIC ENGINEERING MECHANICS
www.elsevier.com/locate/probengmech

Stochastic analysis of failure of earth structures

Michal Šejnoha*, Jiří Šejnoha, Marie Kalousková, Jan Zeman

Department of Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague, Thákurova 7, 166 29 Prague 6, Czech Republic

Received 16 March 2006; received in revised form 27 September 2006; accepted 8 November 2006
Available online 22 December 2006

Abstract

Uncertainties in material data are a common inconvenience we face when working in the area of geotechnical engineering. Elements of mathematical statistics then often become a valuable tool for allowing reasonable predictions of the behavior of complex material systems. Such an approach is advocated in this paper through two representative examples. Stochastic analysis of failure of dump slopes (tailings) is addressed first, promoting the entire distribution function as an indispensable source of information to assess the quality of the structural system from the stability perspective. The general concept of probability of failure is then revisited in conjunction with time dependent failure of earth structures impaired by a gradual change in the level of ground water table. A conceptual assessment of the instantaneous failure rate, particularly when combined with in situ measurements, is offered as a valuable tool for the design engineer to foresee sudden and catastrophic failures.

Keywords: Factor of safety; Probability distributions; Simulation methods; Latin hypercube sampling; Failure probability; Failure rate

1. Introduction

A deterministic approach to the analysis of engineering problems is justified merely in cases where the input data is such that the expected results are assumed to be more or less "unique". Unfortunately, such a situation hardly ever occurs in practice, especially in geotechnical applications where the input data often acquires a label of being uncertain rather than well defined. An inability in describing the material by a single set of parameters thus naturally opens the way to the field of statistics. Accepting input data with random characteristics clearly requires a considerably different approach to the solution of the problem of stability of earth slopes when compared to a purely deterministic analysis. A probabilistic approach to the modeling of failure of earth structures is the prominent topic of this paper.

This subject, although still under continuous expansion, has earned considerable attention in the literature, particularly in the last two decades. An extensive overview of the field of statistics applied to various sub-disciplines of soil mechanics, including the concept of random loading, random material properties and random boundaries is presented in [1]. Random character of internal boundaries separating regions with different material properties has been addressed, e.g. in [2] within the framework of the probabilistic finite element method. See also [3,4] for preliminary works on this subject. This approach, generally classified as non-statistical, finds its roots in the perturbation technique, which allows for taking into account probabilistic characteristics through Taylor expansions of respective random variables. The method of second order moments used for example in the nonlinear analysis of embankments with spatially varying soil properties [5] also falls within this category. However, with a higher degree of nonlinearity these methods become less attractive owing to the need of higher order derivatives to be introduced into the Taylor series, which in turn are difficult to obtain numerically with the desired accuracy. In view of this, the authors often turned their attention to so-called statistical methods generally based on repeated simulations. Among others the direct Monte Carlo technique still appears as the most widely spread simulation method in various sub-disciplines of soil mechanics including soil dynamics [6], evaluation of foundation bearing capacity [7–9], slope stability

* Corresponding author. Tel.: +420 2 2435 4494; fax: +420 2 2431 0775.
E-mail addresses: sejnom@fsv.cvut.cz (M. Šejnoha), sejnoha@fsv.cvut.cz (J. Šejnoha), kalouskova@fsv.cvut.cz (M. Kalousková), zemanj@cml.fsv.cvut.cz (J. Zeman).

0266-8920/$ - see front matter
doi:10.1016/j.probengmech.2006.11.003

problems [10,5,11–14], and many others. Although appealing to most practical engineers, this method becomes inadmissibly expensive particularly when applied in conjunction with the analysis of real world structures having a high degree of material, geometrical and nonlinear complexity. To provide a suitable alternative Fenton and Griffiths [13] introduced a local homogenization approach employing harmonic averaging of spatially varying soil properties. Regardless of the method used, the key result common to all investigations suggests that including uncertainties in the analysis of earth structures considerably changes the expected response when compared to the pure deterministic analysis and generally predicts a reduction in the bearing capacity of the analyzed structure.

Unlike many applications, where the main concern is devoted to naturally occurring geological materials with very high spatial variability in their structure, this contribution revisits the field of material uncertainties of man-made structures such as dump slopes (tailings) or dam structures, which also deserve considerable attention primarily in view of several recently reported failures. The random field approach typical for former material systems is then often replaced by the random variable approach or a combination of the two in a certain simplified manner as demonstrated in this paper through the solution of two representative problems. A simple non-softening analysis of slope stability problems with an elastic–perfectly plastic material model of the Mohr–Coulomb type is adopted to provide the basic computational framework. Essential steps of this analysis in conjunction with the finite element method are briefly reviewed in Section 2.

At minimum the failure analysis of earth structures when presented in the framework of Mohr–Coulomb failure criterion [15] requires the knowledge of at least four parameters, which are often assumed to have a random character. They are essentially random functions, which vary both in time t and space [16]. The material parameters subjected to random variation are the self-weight $\gamma = \gamma(x, y)$, effective strength parameters $c = c(x, y)$ and $\varphi = \varphi(x, y)$, and pore pressure $p = p(x, y, t)$ [17]. Apart from uncertainties in their expected values these parameters often show some degree of correlation, both spatial and material. A proper treatment of these effects thus clearly requires an introduction of the concept of failure in a stochastic rather than deterministic framework. Fundamental principles of such an approach, including the discussion on simulations, sampling and minimum sample size, are summarized in Section 3 and further applied to the solution of two example problems. While the first problem presented in Section 4.1 merely serves to provide the route for a realistic prediction of the probability of failure including material and spatial correlation of assumed random variables, the second problem discussed in Section 4.2 extends the proposed methodology to a reliable prediction of the evolution of the probability of failure as a function of time.

To present the general solution strategies in a manner that appeals primarily to the design engineers inevitably calls for certain simplifications in the theoretical formulation. Attention certainly deserves an application of the correlation matrix in place of the consistent correlation theory when addressing the statistical dependence of random parameters between individual stochastic quasi-homogeneous regions as assumed in the example of a dump slope, see Section 4.1.2. Note that sufficiently fine subdivision brings this approach closer to the classical solution based on random fields [12,13]. Solving the second problem further draws on the possibility of estimating the time variation of the factor of safety. This is however the only assumption promoting a general applicability of the proposed methodology. One particular example is then presented in Section 4.2.2.

2. Finite element method in stability analysis of earth structures

The solution strategy of the slope stability problem when introduced in conjunction with the finite element method draws on the assumption that the forces generated by the self-weight represent the only source of loading and are applied in a single increment to an initially stress-free slope. It has been argued in [12,14,13] that the predicted factor of safety derived from simple plasticity models (Mohr–Coulomb, Drucker–Prager) under drained boundary conditions, as is assumed in the present study, is insensitive to the form of gravity application, thus justifying the use of a gravity *turn-on* procedure.

Such a statement, however, inevitably suggests some trafficking with the limit equilibrium methods. Application of the classical method of slices combined with spatial correlation and the Latin Hypercube Sampling (LHS) method was reported in [18]. Two specific approaches were proposed to address the usually observed progressive failure in earth slopes. The first approach treated material parameters with their expected values known at a finite number of sampled points as components of a random vector. Some method of interpolation (e.g. isoparametric finite elements) is therefore needed to incorporate spatial correlation. Unfortunately, this approach fails to be effective when working with a large number of measured points (certainly advantageous when mapping a construction site). The second approach thus introduces average random processes and assumes homogenized properties within a given segment. Fluctuation of material properties is then assumed along the mean slip surface. This approach proved to be particularly efficient when estimating the probability of the onset of failure and its subsequent progression. A similar problem was also addressed in [19].

Although simple from the implementation point of view, this approach suffers from some intrinsic drawbacks. In particular, being in the hands of the limit equilibrium approach it is generally assumed that the failure occurs when the critical state condition is generated along an entire “a priori” selected slip surface. Such a condition is expected to occur suddenly without any prior knowledge about the evolution of stresses and displacements preceding the failure. The finite element method allows, on the other hand, for keeping the track of the gradual evolution of the slip surface, manifested for example by the localized zone of equivalent plastic strain, with the level of the applied load thus giving the location of the onset of failure