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Analytic solutions of problem about a circular hole with a straight crack in one-dimensional hexagonal quasicrystals with piezoelectric effects

J. Yang [a,b], X. Li [a,*]

[a] School of Mathematics and Computer Science, Ningxia University, 750021 Yinchuan, PR China
[b] Institute of Ethnic Preparatory Education, Ningxia University, 750002 Yinchuan, PR China

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ABSTRACT

The anti-plane shear problem about a circular hole with a straight crack in one-dimensional (1D) hexagonal quasicrystals (QCs) with piezoelectric effects is investigated by means of the complex variable function method and using the technique of conformal mapping. The analytic solutions of the stress intensity factors (SIFs) and the electric displacement intensity factors (EDIFs) with electrically impermeable and permeable conditions were obtained, and these solutions have an important theoretical significance for the engineering application of QCs materials. When the circle radius tends to zero, the present results can be reduced to the cases of the Griffith crack. In the absence of the phason field, the obtainable results in this paper agrees well with the results for piezoelectric materials. Numerical analysis is then conducted to discuss the influences of geometric parameters and applied mechanical/electric loads on the field intensity factors and energy release rate.

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1. Introduction

As a class of smart materials, piezoelectric materials have been widely used in adaptive microelectro-mechanical systems such as sensors, actuators, and transducers due to a strong coupling characteristic between elastic and electric behaviors [1,2]. Now, piezoelectric sensors, actuators, and transducers of various configurations can be manufactured for specified functions [3]. For example, in signal processing applications, with the aid of excitation or reception of the surface acoustic waves, an interdigital transducer is a thin piezoelectric layer bonded perfectly on a elastic substrate. And on the surface of the piezoelectric film, an array of electrodes is arranged according to different patterns [4].

Shechtman was awarded the Nobel Prize of Chemistry 2011 due to the discovery of QCs in 1984, which has arisen the great interest on the structure and material again [5]. The theoretical frame of QCs comes from physical research which has been done by some researchers [6,7]. Based on Landau–Anderson symmetry-breaking, the phason as a new elementary excitation was introduced in addition to the well known phonon. QCs possess unique atomic structures with perfect long-range positional order with noncrystallographic rotational symmetry. Both the mass-density-wave and the unit-cell described in higher dimensional space can be used to decipher their atomic structures [8,9]. According to the cut-and projection method, a 3D quasilattice can be obtained by selected projection of the respective 6D periodical lattice [10,11]. Therefore, there are two kinds of displacement fields in elasticity. One is a phonon displacement field, which is the same as the displacement field of usual crystals, and whose gradient describes the change in shape and volume macroscopically. The other is new and named as a phason displacement field, which is diffusive due to the elementary excitation associated with the phason mode and describes the local rearrangements of the unit-cells. Experiments have shown that QCs are quite brittle [12] and the defects of quasicrystalline materials have been observed [13]. When quasicrystalline materials are subjected to mechanical stresses in service, the propagation of flaws or defects produced during their manufacturing process may result in premature failure of these materials. Therefore, the study of crack problem of quasicrystalline materials is meaningful both in theoretical and practical applications. Fan [14] presented the mathematical theory of quasicrystalline elasticity. Using this theory, a straight dislocation and a moving screw dislocation in 1D hexagonal QCs were addressed by Li and his coauthors [15,16]. It is well known that the complex variable method initially developed by Muskhelishvili is an effective method for solving various elasticity and defect problems [17]. Therefore, some workers developed the complex variable method to solve defect problems

* Corresponding author. Tel.: +86 954 2079018.
E-mail address: li_x@nxu.edu.cn (X. Li).

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of QCs. For 1D hexagonal QCs many efforts have been made in the fields of the mechanic involving the elasticity and defects [18–22]. Wang and Pan [23] studied some typical defect problems in octagonal QCs and derived the general solutions of elastic fields by means of the differential operator theory and the complex variable method. Some Reducing the boundary value problem to the Riemann–Hilbert problem of periodic analytic functions, Shi [24] obtained the closed-form solutions of collinear periodic cracks and/or rigid inclusions of antiplane sliding mode in 1D hexagonal QCs.

Although many crack problems of 1D hexagonal QCs have been investigated, but the paper mentioned before discussed various elasticity and defect problems of QCs. To our knowledge, the QCs with piezoelectric effects researches are very little, especially the study of QCs piezoelectric properties. Analytic solutions of two collinear fast propagating cracks in a symmetrical strip of 1D hexagonal piezoelectric QCs have been studied [25]. The present paper is devoted to investigating the elastic problem of a circular hole with a straight crack in 1D hexagonal QCs with piezoelectric effects by means of complex variable function method with conformal mapping. Two kinds of crack surface conditions, i.e. electrically impermeable and permeable are adopted. The exact solutions of SIFs for the phonon field and the phason field, and the EDIFs are obtained respectively, which are useful in practice. When the circle radius tends to zero, the present results can be reduced to the cases of the Griffith crack. Furthermore, in the absence of the phason field, the exact solutions of the field intensity factors presented in this paper can be degenerated into the corresponding results of piezoelectric materials.

2. Basic theory

The generalized Hooke's law of 1D hexagonal QCs with piezoelectric effects, whose period plane is the (x_1, x_2)-plane and whose quasiperiodic direction is the x_3-axis, is given by Wang and Pan [23]

$$
\begin{aligned}
&\sigma_{11} = C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} + R_1 w_3 - e^1_{31}E_3,\\
&\sigma_{22} = C_{12}\varepsilon_{11} + C_{11}\varepsilon_{22} + C_{13}\varepsilon_{33} + R_1 w_3 - e^1_{31}E_3,\\
&\sigma_{33} = C_{13}\varepsilon_{11} + C_{13}\varepsilon_{22} + C_{33}\varepsilon_{33} + R_2 w_3 - e^1_{33}E_3,\\
&\sigma_{23} = \sigma_{32} = 2C_{44}\varepsilon_{32} + R_3 w_2 - e^1_{15}E_2,\\
&\sigma_{31} = \sigma_{13} = 2C_{44}\varepsilon_{31} + R_3 w_1 - e^1_{15}E_1,\\
&\sigma_{12} = \sigma_{21} = 2C_{66}\varepsilon_{12},\\
&H_1 = 2R_3\varepsilon_{31} + K_2 w_1 - e^2_{15}E_1,\\
&H_2 = 2R_3\varepsilon_{32} + K_2 w_2 - e^2_{15}E_2,\\
&H_3 = R_1(\varepsilon_{11} + \varepsilon_{22}) + R_2\varepsilon_{33} + K_1 w_3 - e^2_{33}E_3,\\
&D_1 = 2e^1_{15}\varepsilon_{31} + e^2_{15}w_1 + \in_{11}E_1,\\
&D_2 = 2e^1_{15}\varepsilon_{32} + e^2_{15}w_2 + \in_{11}E_2,\\
&D_3 = e^1_{31}(\varepsilon_{11} + \varepsilon_{22}) + e^1_{33}\varepsilon_{33} + e^2_{33}w_3 + \in_{33}E_3,
\end{aligned}
\tag{1}
$$

where $\sigma_{ij},\ H_j(i,j = 1,2,3)$ are the phonon and phason stress components; $E_j,\ D_j(j = 1,2,3)$ are the electric fields and the electric displacements; $\varepsilon_{ij},\ w_j(i,j = 1,2,3)$ are the phonon and phason strains; $C_{11},\ C_{12},\ C_{13},\ C_{33},\ C_{44},\ C_{66}$ are elastic constants in the phonon field, $C_{66} = \frac{C_{11}-C_{12}}{2}$; $K_1,\ K_2$ are elastic constants in the phason field; $R_i(i = 1,2,3)$ are phonon–phason coupling elastic constants; $e^1_{ij},\ e^2_{ij}(i = 1,3, j = 1,3,5)$ are piezoelectric coefficients; $\in_{11},\ \in_{33}$ are dielectric coefficients.

The strain–displacement and electric field–electric potential relations are given by

$$
\varepsilon_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j),\ w_j = \partial_j v,\ E_j = -\partial_j \phi,\ (i,j = 1,2,3), \tag{2}
$$

where u_i and v denote the displacements of phonon field and phason field, φ is the electric potential. Here we have used the tensor notation and $\partial_j u_i = \frac{\partial u_i}{\partial x_j}$, the same hereafter.

In the absence of body force and electric charge density, the equilibrium equations are

$$
\begin{aligned}
&\partial_1\sigma_{11} + \partial_2\sigma_{12} + \partial_3\sigma_{13} = 0,\\
&\partial_1\sigma_{21} + \partial_2\sigma_{22} + \partial_3\sigma_{23} = 0,\\
&\partial_1\sigma_{31} + \partial_2\sigma_{32} + \partial_3\sigma_{33} = 0,\\
&\partial_1 H_1 + \partial_2 H_2 + \partial_3 H_3 = 0,\\
&\partial_1 D_1 + \partial_2 D_2 + \partial_3 D_3 = 0.
\end{aligned}
\tag{3}
$$

For the anti-plane shear problem in which all field variables are independent of x_3. In this case, we have the following deformation geometrical equations

$$
\partial_3 u_i = 0, \partial_3 v = 0, \partial_3\sigma_{ij} = 0, \partial_3 H_i = 0, \partial_3 D_i = 0, i,j = 1,2,3 \tag{4}
$$

Substitution Eq. (4) into Eqs. (1)–(3) leads to one is a plane elasticity problem of general crystals, which can be solved by the route of linear elastic theory [17]. We do not discuss it here. And the other is an anti-plane phonon–phason coupling elasticity problem as follows:

$$
\begin{aligned}
&\sigma_{23} = \sigma_{32} = 2C_{44}\varepsilon_{32} + R_3 w_2 - e^1_{15}E_2,\\
&\sigma_{13} = \sigma_{31} = 2C_{44}\varepsilon_{31} + R_3 w_1 - e^1_{15}E_1,\\
&H_1 = 2R_3\varepsilon_{31} + K_2 w_1 - e^2_{15}E_1,\\
&H_2 = 2R_3\varepsilon_{32} + K_2 w_2 - e^2_{15}E_2,\\
&D_1 = 2e^1_{15}\varepsilon_{31} + e^2_{15}w_1 + \in_{11}E_1,\\
&D_2 = 2e^1_{15}\varepsilon_{32} + e^2_{15}w_2 + \in_{11}E_2,\\
&\partial_1\sigma_{31} + \partial_2\sigma_{32} = 0,\\
&\partial_1 H_1 + \partial_2 H_2 = 0,\\
&\partial_1 D_1 + \partial_2 D_2 = 0,\\
&\varepsilon_{3j} = \varepsilon_{j3} = \frac{1}{2}\partial_j u_3,\ w_j = \partial_j v,\ E_j = -\partial_j\phi,\ j = 1,2.
\end{aligned}
\tag{5}
$$

From Eq. (5), one obtains

$$
\begin{aligned}
&C_{44}\nabla^2 u_3 + R_3\nabla^2 v + e^1_{15}\nabla^2\phi = 0,\\
&R_3\nabla^2 u_3 + K_2\nabla^2 v + e^2_{15}\nabla^2\phi = 0,\\
&e^1_{15}\nabla^2 u_3 + e^2_{15}\nabla^2 v - \in_{11}\nabla^2\phi = 0,
\end{aligned}
\tag{6}
$$

where $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$ is the two dimensional Laplace operator. Under the condition of $\begin{vmatrix} C_{44} & R_3 & e^1_{15} \\ R_3 & K_2 & e^2_{15} \\ e^1_{15} & e^2_{15} & -\in_{11} \end{vmatrix} \neq 0$, Eq. (6) can be written as

$$
\nabla^2 u_3 = 0,\ \nabla^2 v = 0,\ \nabla^2\phi = 0. \tag{7}
$$

According to the theory of complex variable function, $u_3,\ v$ and ϕ can be represented as the real part or the imaginary part of three analytic functions $\varphi_i(z)(i = 1,2,3)$. We assume that

$$
u_3 = Re\varphi_1(z),\ v = Re\varphi_2(z),\ \phi = Re\varphi_3(z), \tag{8}
$$

where $z = x_1 + ix_2,\ i = \sqrt{-1}$.

3. Solution of elastic field

Considering a problem about a circular hole with a crack along the quasi-periodic direction (x_3 direction) in a 1D hexagonal QCs. R is the circle radius and L is crack length. The solid is subjected to uniform remote anti-plane shear and in-plane electric field loading, as shown in Fig. 1.

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