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Stress intensity factor for hypocycloidal hole in finite plate

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A B S T R A C T

In this paper, the general solution for the stress intensity factor at cusp points of hypocycloidal hole in anisotropic finite plate subjected to in-plane loading is presented. The stress intensity factors at cusp points of hypocycloidal hole is derived using complex variable approach. The stress functions are represented in terms of infinite power series of complex numbers and the constants of the series are derived using boundary collocation method. The effect of plate size, material properties, hole geometry and loading angle is also studied. The results obtained through present method are compared with that of the literature.

1. Introduction

The plate-like structural components made of composite materials are very commonly used nowadays in different engineering applications especially in automobile and aerospace industries. The plates are generally provided with different shapes of holes/cutouts for certain operational needs. The discontinuities may evolve due to chemical or environmental effect. These cutouts or discontinuities work as stress risers in the plate subjected to loading.

The solutions of stress analysis of plate with different types of discontinuities have been addressed by many researchers since more than a century. Lekhnitskii [1], Savin [2], Ukadgaonker and Rao [3], Ukadgaonker and Kakhandki [4], Simha and Mohapatra [5], Razaeepazhand and Jafari [6], Batista [7], Sharma [8–10], Patel and Sharma [11], Daoust and Hoa [12], etc. have provided the solutions for the stresses around various shapes of the hole using Muskhelishvili's [13] complex variable approach for isotropic/anisotropic infinite plates where the load is applied at remote boundary.

The problem of stresses around the hole infinite plate is addressed by some researchers like Ogonowskii [14], Newman [15] Xu et al. [16], Zheng and Xu [17], Madenci et al. [18], Lin and Ko [19], Woo and Chan [20], Pan et al. [21], etc. using the boundary collocation method in conjunction with a complex variable approach. These solutions provide the stress concentration factor around circular, elliptical or rectangular hole in finite isotropic/anisotropic plate subjected to in-plane loading. Chauhan and Sharma [22], recently presented the solution of stresses in finite anisotropic plate weakened by rectangular hole.

The hole or discontinuity generated through chemical or environmental effect, generally have sharp corners where evaluation of stress intensity factor (SIF) at the sharp corner is the major concerns. The solution of stress intensity factors for hypocycloidal, aerofoil and symmetric lip type of hole in infinite isotropic plate are proposed by Chen et al. [23], Nik Long and Yaghobifar [24] using complex variable approach. The solution of stress intensity factor for a hypocycloidal hole in infinite anisotropic plate is also presented recently by Sharma and Dave [25]. Very recently, Sharma [26] has given the solution for stress intensity factors for hypocycloidal hole as a special case of hypotrochoidal hole in infinite isotropic plate.

The solution of stress intensity factors in finite plate is presented by some researchers like Bowie and Freese [27], Bowie and Neal [28], Kobayashi et al. [29], Gyekenyesi and Mendelson [30], Fu and Zhang [31], Peng and Jones [32], etc. for cracks or crack emanating from a hole. The review of the literatures identifies that the solution of stress intensity factor for hypocycloidal hole in finite anisotropic plate is still not addressed.

An attempt is made here to present a general solution for the stress intensity factor for a hypocycloidal hole in the finite anisotropic plate subjected to in-plane loading. The complex variable approach in conjunction with boundary collocation method is used to derive the series form of complex stress functions. The effect of plate size, material properties, hole geometry and loading angle on SIFs is presented for finite laminated plate with centrally located hypocycloidal hole with cusp.

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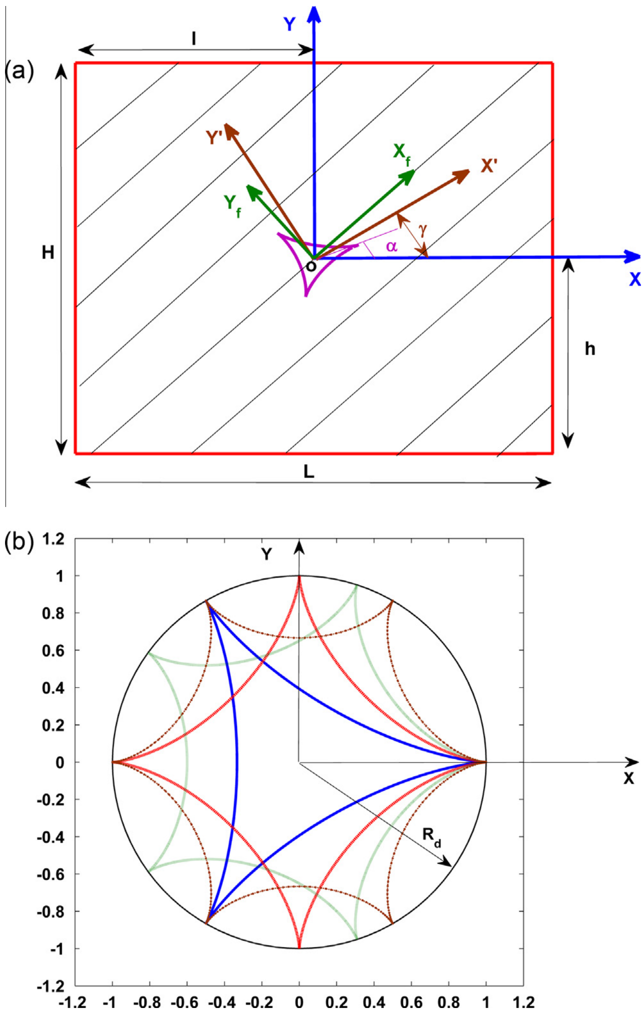

Fig. 1. Geometry of finite plate and hypocycloidal hole with cusp.

2. Analytical formulation

A finite plate (Fig. 1(a)) with hypocycloidal hole is subjected to in-plane loading on the edges of the plate. The geometry of the plate is defined with respect to the global coordinate system *XOY*, defined at the center of the hole. The dimensions *l*, *h*, *L* and *H* are normalized with respect to R_d (Fig. 1(b)) where *l* and *h* show the normalized distance of left and bottom edges of the plate respectively, and *L* and *H* show the normalized length and height of the plate respectively. The load is applied in *X′OY′* coordinate system at γ angle from *XOY*. The fibers of a composite lamina are oriented at β angle in X_fOY_f coordinate system. α is the orientation angle of the hole.

Table 2
Comparison of SIF in Glass/Epoxy$[0/90]_s$ infinite plate, $\lambda = 0, \gamma = 90^\circ$.

m	Cusp location (°)	Present method (*L* = *H* = 100)		Sharma and Dave [25]	
		K_I/K_0	K_{II}/K_0	K_I/K_0	K_{II}/K_0
3	0	1.324	0	1.3333	0
	120	0.332	−0.283	0.3333	−0.2866
	240	0.332	0.283	0.3333	0.2866
4	0	1.533	0	1.5000	0
	90	−0.039	−0.001	0	0
	180	1.518	0	1.5000	0
	270	−0.039	0.001	0	0
5	0	1.654	0	1.6000	0
	72	0.149	0.236	0.1528	0.2334
	144	1.017	−0.374	1.0472	−0.3777
	216	1.017	0.374	1.0472	0.3777
	288	0.149	−0.236	0.1528	−0.2334

Table 3
Mode I SIF at 0° cusp in isotropic finite plate ($\lambda = 0, \gamma = 90^\circ, \alpha = 0, R_d = 1$).

Plate size (*L* = *H*)	*m*	Present method	ANSYS
6	3	1.50	1.5912
	4	2.38	2.2650
	5	2.39	2.2784
8	3	1.43	1.4947
	4	2.13	2.0664
	5	2.13	2.0731
10	3	1.39	1.3959
	4	2.03	1.9269
	5	2.09	1.9236

In the 2D theory of elasticity the stress components for anisotropic media are written in terms of Muskhelishvili's [13] complex stress function as,

$$\sigma_x = 2Re\left[\sum_{j=1}^{2}\mu_j^2\phi_j'(z_j)\right]$$
$$\sigma_y = 2Re\left[\sum_{j=1}^{2}\phi_j'(z_j)\right] \quad (1)$$
$$\tau_{xy} = 2Re\left[\sum_{j=1}^{2}\mu_j\phi_j'(z_j)\right] \quad (j = 1, 2)$$

where $\phi_j(z_j)$ are Muskhelishvili's [13] complex stress functions, $\phi_j'(z_j)$ are the first derivatives of complex stress functions $\phi_j(z_j)$, $z_j = x + \mu_j y$ and μ_j are the complex constants of anisotropy. These constants are the roots of the characteristic equation obtained by applying the generalized Hooke's law, Airy's stress functions and strain–displacement compatibility conditions to anisotropic plate (Lekhnitskii [1]). The constants of anisotropy depend on material properties, fiber orientation and stacking sequence.

Table 1
Comparison of SIF in isotropic infinite plate for *m* = 3.

Loading condition	Cusp location (°)	Present method (*L* = *H* = 100)		Sharma and Dave [25]		Chen et al. [23]		Nik and Yaghobifar [24]	
		K_I	K_{II}	K_I	K_{II}	K_I	K_{II}	K_I	K_{II}
$\lambda = 0, \gamma = 90^\circ$	0	2.0467	0	2.0467	0	2.0466	0	2.0467	0
	120	0.5112	0.835	0.5117	0.8862	0.5116	0.8862	0.5117	0.8862
	240	0.5112	0.835	0.5117	0.8862	0.5116	0.8862	0.5117	0.8862
$\lambda = 0, \gamma = 0$	0	0	0	0	0	0	0	0	0
	120	1.6308	0.887	1.5358	0.8862	1.5349	0.8862	1.5358	0.8862
	240	1.6308	0.887	1.5358	0.8862	1.5349	0.8862	1.5358	0.8862