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Computation of the J_k-integrals for bimaterial interface cracks using boundary element crack shape sensitivities

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ABSTRACT

This paper presents a new algorithm for the efficient evaluation of J_k-integrals for cracks between bonded homogeneous and isotropic materials using the boundary element crack shape sensitivities (BECSS). The flexibility of this novel method allows for analysis of both curved and straight interface cracks. In contrast to the available algorithms, the present method does not require stress analysis at a series of internal points around the crack or employment of an auxiliary equation. For an interface crack, the J_1-integral is the strain energy release rate (SERR) or the derivative of the total potential energy with respect to the crack length extension. Although the J_2-integral shows an oscillatory type behaviour and is non-existent at the crack tip, it can also be evaluated by direct differentiation of the structural response. It is well-known that a bimaterial interface crack induces both opening and shearing behaviour even for a single mode loading. Here it is shown that the computed J_k can be used to decouple and estimate the stress intensity factors (SIFs). Here, three example problems are analysed and their J_k values are presented which are in excellent agreement with the corresponding analytical results. Each case includes the contribution to J_2 by the jump of displacement derivatives across the interface and the strain energy density discontinuity on the crack surfaces and interface region.

1. Introduction

Composite materials have endless applications in a variety of industries including aerospace, automobile, naval and electronics. In a composite structure consisting of two or more materials with various properties such as fibre reinforced laminated composites or multilayered electronic devices, failure is more likely to initiate at interfaces. Williams [1] was the first scientist to discover that the stress field along an interface crack between two dissimilar elastic materials is not only singular, but also has an oscillatory behaviour of type $r^{-\frac{1}{2}+i\varepsilon}$ where r is the radial distance from the crack tip and ε is a bimaterial constant. This shows that when r approaches zero, the displacements and stresses change sign indefinitely and there is an interpenetration of two crack faces near the crack tip which is not physically possible. As suggested by several researchers, for most practical crack sizes and materials, this zone of contact is extremely small and typically of the order of a few nanometers. In linear elastic interfacial fracture mechanics (LEIFM), the overlap between crack faces at the crack tip is usually ignored. However, an asymptotic field characterizing the stress and strain is usually employed in the vicinity of the crack tip.

William's work was followed by the studies carried out by Rice and Sih [2], Erdogan [3,4] and England [5]. Following, their pioneering research, a variety of algorithms have been developed based on LEIFM and in conjunction with the boundary element method (BEM), finite element method (FEM) or analytical method. These methods are based on the virtual crack extension, M-integral, interaction integral, complex variable, numerical manifold, element free Galerkin, extended finite element (XFEM) and analytical mode separation [6–15].

At present, the most common method used in industry and by academia for solving fracture mechanics of homogeneous structures is the J_1-integral [16,17] in conjunction with BEM or FEM. The J_1-integral is the Rice's path independent integral. This method was first developed by Rice to characterize fractures for two-dimensional structures with linear or nonlinear elastic material behaviour. Although the J_1-integral with BEM reduces computational time, it still requires time-consuming stress analysis at a series of internal points around the crack.

For elastic problems, the J_1-integral is the SERR per unit of the crack extension. In conjunction with the FEM or BEM, it is possible to directly evaluate the sensitivities of the total strain energy where the crack length is being treated as the shape variable. Ref. [18] presents the novel application of the BECSS for the evaluation of the J_1-integral in anisotropic materials where the crack of

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Nomenclature

a	half crack length or crack half chord length
a_1, a_2	local coordinates at the crack tip
C_{jk} (P)	limiting value of the surface integral of T_{jk} (P,Q)
E	Young's modulus
F_i	shape functions for the continuous quadratic elements
f_i	shape functions for the discontinuous quadratic elements
E_P	potential energy
G	shear modulus
I_{Sc}	integral involving strain energy density and strain discontinuities
J_k	$(k = 1,2)$, J-integral
K_1, K_2	modes 1 and 2 stress intensity factors, respectively
N_i $(i = 1,3)$	geometrical nodes
Nc_i $(i = 1,3)$	collocation nodes
n_1, n_2	direction cosines of the unit outward normal vector to the surface of the elastic body
P	load point at the surface of the elastic domain
Q	field point at the surface of the elastic domain
r	small radial distance from the crack tip
R	radius of the circular inclusion
S	domain boundary
T_{jk} (P,Q)	jth component of the traction vector at point Q due to a unit point load in the kth direction at P
t_j	traction vector
U_{jk}	jth component of the displacement vector at point Q due to a unit point load in the kth direction at P
u_j	displacement vector
W	strain energy density
x_i	rectangular Cartesian coordinates (Global)
ε	bimaterial constant
ν	Poisson's ratio
θ_0	half the central crack angle
σ_{ij}	stress tensor
σ_{x0}, σ_{y0}	normal stress components in the x- and y-directions, respectively
η	ratio E_1/E_2
τ_0	shear stress component
ψ_i	$\frac{3-\upsilon_i}{1+\upsilon_i}$ plane stress, $3-4\upsilon_i$ plane strain
ζ_1, ζ_2	coordinates of load point
ζ	natural coordinate

arbitrary geometric shape, straight or curved, was treated as the shape design variable. Since fracture mechanics parameters were evaluated by direct differentiation of the structural response, the BECSS method is computationally more accurate and efficient than the J_1-integral method.

For fracture of in-plane mixed mode cracks in homogeneous structures, the J_1-integral is related to a combination of SIF values, due to the different fracture modes. The decomposition method is the most popular technique for the evaluation of SIFs [19]. An alternative for decoupling the SIFs is to evaluate the J_2-integral which not only involves the computation of stresses and strains at a series of internal points around the crack but also the evaluation of highly singular integrals over the crack surfaces. In Ref. [20] this deficiency is overcome by direct evaluation of J_2 using BECSS where a small region around the crack tip is treated as the shape design variable. It is shown that the derivative of the total potential energy with respect to the transverse direction of the crack is not the J_2-integral. However, by addition of an integral, involving the strain energy density discontinuity, to this derivative the J_2-integral can be efficiently evaluated. That study focused on isotropic and homogeneous materials. For the sake of validation the selected case studies with known analytical solutions were employed where for each crack shape and loading condition, the corresponding values of J_1, J_2 and also the contribution to J_2 from the strain energy density discontinuity were presented.

In Ref. [21], using the BECSS of multi-region domains, coupled with an optimization algorithm and an automatic mesh generator, the crack kink angle and crack propagation path in anisotropic and homogeneous elastic solids, based on the maximum SERR criterion, were predicted. In contrast to the J_1-integral method, the computation of stresses and strains at a series of internal points during the automatic incremental crack procedure was not required. Therefore, the method was more accurate and efficient. The prediction of the crack propagation path of a central slant crack in a titanium plate subject to tension was in very good agreement with the corresponding experimental results published elsewhere. The findings confirmed the simplicity, accuracy and flexibility of the method which can be applied to both curved and straight cracks.

Here, the J_k-integrals for interface cracks between bonded homogeneous, isotropic and dissimilar materials are obtained using the BECSS. It is demonstrated how a good estimation of SIFs can be made using the computed J_k values. Three example problems with straight or curved interface cracks are analysed and their corresponding J_k values are presented. The results include the contribution to J_2 from the jump of displacement derivatives or strain across the interface and also the strain energy density discontinuity on the crack surfaces and interface region.

2. Review of the boundary element crack shape sensitivity analysis

The BEM is based on the unit load solutions in an infinite body, known as the fundamental solutions; used with the reciprocal work theorem and appropriate limit operations. The Boundary Integral Equation (BIE) of the BEM for homogeneous and isotropic materials is an integral constraint equation relating boundary tractions (t_j) and boundary displacements (u_j) and it may be written as [22]

$$C_{ij}u_j(P) + \int T_{ij}(P,Q)u_j(Q)ds(Q) = \int U_{ij}(P,Q)t_j(Q)ds(Q) \quad i,j = 1,2 \tag{1}$$

$P(\zeta_1,\zeta_2)$ and $Q(x_1,x_2)$ are the load and field points, respectively. Following the numerical integration, BIE can then be reduced to a set of simultaneous linear equations and be solved. The constant C_{ij} depends on the local geometry of the boundary at P, whether it is smooth or sharp. For a general crack problem involving a homogeneous body with mixed mode deformation or an interface crack between bonded dissimilar materials, the domain may be divided into several subregions in which the crack faces coincide with the boundaries of the subregions [23]. The BIE can then be employed for each subregion (L), in turn. Then appropriate continuity and equilibrium conditions are applied at the common subregion interface before the linear algebraic equations are solved.

Shape sensitivity analysis (SSA) is the calculation of quantitative information on how the response of a structure is affected by changes in the variables that define its shape. SSA is the fundamental requirement for shape optimization. The BEM, being a surface oriented technique, is well suited for shape and topology optimization problems, in particular for SSA [24–26]. In order to obtain sensitivities of the structural response with respect to a