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# Effect of boundary conditions on the interfacial fracture behavior: Circular arc antiplane crack model for an annular sector bilayer plate

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ABSTRACT

In this paper, the problem of an arc-shaped interface crack in an annular sector bilayer plate under axial shear loading is analyzed. By considering sixteen frequently encountered boundary conditions, the Fourier transform method is used to reduce the mixed boundary value problem to a singular integral equation associated with a mode-III crack, from which the stress intensity factors are obtained numerically by the Lobatto–Chebyshev quadrature technique. To demonstrate the computational accuracy of the present method, convergence and validation studies are carried out. The computational results of stress intensity factors show that there exists a coupled effect between geometrical and physical parameters on the interfacial fracture behavior clearly, and the effect of the stiffness ratio on the interfacial fracture behavior depends on the selection of the circumferential edges.



## 1. Introduction

Circular, annular or sector flat plates are widely regarded to have light weight, high load-carrying capacity, significant economy and great technological effectiveness [1]. Owe to these superiorities, such plate has been widely used in engineering, such as architecture, hydraulics, aerospace, marine and other various industrial equipments [2,3]. For example, bridge decks, thrust bearing plates, turbine disks and mechanical clutches are some typical examples among others [4].

Numerous investigations have been carried out on the mechanics of homogeneous or inhomogeneous plates of circular, annular or sector geometries. Fig. 1 shows a sketch of an annular sector plate with $r_0$ and $r_1$ as the inner and outer radius respectively, and $\gamma$ as the sector angle. Various boundary conditions, such as the simply supported (S), clamped (C) and free (F), have been defined in many studies on the bending, buckling and vibration of annular sector plate [5]. These works showed that the mechanical behavior of the plate is largely affected by these boundary conditions. In this regard, natural frequencies of vibration for a circular plate with different combinations of free, simply-supported, and clamped boundary conditions have been proposed by Irie et al. [6]. Jomehzadeh and Saidi [7] obtained an exact solution for the free vibration of a transversely isotropic sector plate based on the Mindlin's plate theory. In their work, the plate is simply supported radial edges and arbitrary conditions along the circular edge. Comparison of the numerical results indicates that the lowest and highest frequencies correspond to the free and clamped circular edge respectively. Bending analysis of a thick orthotropic sector plate with various loading and boundary conditions demonstrate that the maximum deflection occurs on the free boundary and near to the sliding edge [8]. Based on the linear theory of electroelasticity, Puzyrev [9] investigated the propagation of elastic waves in piezoelectric cylindrical waveguides of circular cross-sections with stress-free and displacement-fixed cylindrical surfaces. Based on a two-parameter foundation model, Alipour et al. [10] displayed the first two mode shapes of a circular plate with free, simply-supported and clamped edge conditions. Both of Pasternak and Winkler parameters were adopted to model the elastic foundation in their analysis. They concluded that, for the simply-supported circular plate, the Winkler and Pasternak coefficients have ignorable effects on the shape modes, while other boundary conditions would yield different results. Thermal buckling analysis of a moderately thick functionally graded annular sector plate was conducted by Saidi and Hasani Baferani [11] numerically. Their results showed that increasing the aspect ratio would lead to a decrease of the critical buckling temperature, except for SSFF boundary condition. Base on the classical plate theory, the natural frequencies of the circular/annular sectorial plates under simply-supported and clamped boundary conditions were

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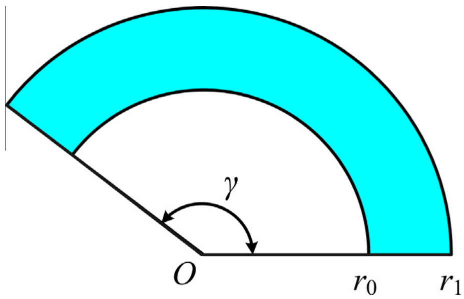


**Fig. 1.** Geometry of an annular sector plate.

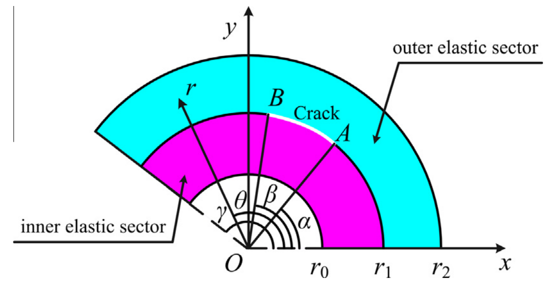


**Fig. 2.** An annular sector bi-layer plate with an arc-shaped interface crack.

obtained by Hosseini-Hashemi et al. [12] employing the differential quadrature method. Xiao et al. [13] exhibited a unified study of finite bending problems for elastic Hill materials. Naderi and Saidi [14] presented an exact analytical solution for buckling analysis of moderately thick functionally graded sector plates based on the Winkler elastic foundation, and observed that the elastic foundation has the most important influence on the critical buckling load with free circular edge to those with the other boundary conditions. Based on the three-dimensional theory of elasticity and the differential quadrature method, axisymmetric static analysis of functionally graded circular and annular plates imbedded in piezoelectric layers were investigated by Alibeigloo and Simintan [15]. In their investigation, the inner and outer edges were assumed with different boundary conditions. Numerical results show that through-the-thickness distribution of transverse shear stress with parabolic form satisfies the surface boundary condition and has the maximum value for the C–F conditions but negligible value for S–S condition. The results of bending of radially functionally graded sector plates by Mousavi and Tahani [5], and their work clearly points out that using functional graded materials in sector plates with mechanical loads has considerable effect on deflection whereas it may increase shear stress depending on the arrangement of boundary conditions. Kiani and Eslami [16] focused on the buckling of a heated functionally graded material annular plate on an elastic foundation and found that only for some special boundary conditions, i.e. the clamped type, the plate remains undeformed in pre-buckling state. This is due to the fact that the temperature distribution does not affect the kinematic boundary conditions corresponding to clamping. Golmakani and Kadkhodayan [17] performed a study on the large deflection behaviors of stiffened annular functionally graded (FG) sector plates under mechanical and thermo-mechanical loadings. The authors explored the effect of boundary conditions on the large deflection behaviors and demonstrated, among other things, that the effect of stiffener depth on decrease of absolute values of deflection is more noticeable in the SSSS rather than the CCCC boundary condition. Shi et al. [18] presented an analytical method for the vibration analysis of annular sector plates with general elastic boundary supports. Their analysis shows that the first natural frequencies are sensitive to the stiffness of the restraining springs, indicating that the change of boundary conditions constitutes an effective way to modify the number of resonant peaks. Ghiasian et al. [19] discussed the bifurcation behaviors of moderately thick heated annular plates. Through the parametric studies, they claimed that the inner and outer boundary conditions can affect the number of nodal diameters, radial buckled shape significantly.

In comparison with the studies mentioned above, however, the fracture analysis for circular, annular or sectorial laminated plate received little attention. Laminated composites are found to be an efficient way to obtain high mechanical strength due to a strong strain coupling between the layers. Problems of composite finite wedges under anti-plane shear applied on a circular arc were analyzed by Chen et al. [20]. Shahani and Ghadiri [21] investigated the anti-plane shear deformation of an anisotropic sector with a radial crack by dual integral equations. In their work, the traction–traction boundary conditions are imposed on the radial edges and the traction-free condition was considered on the circular segment of the sector. In such composites, the interfaces are important for load transfer but meanwhile prone to damages. Arc-shaped cracks are often encountered in laminated circular, annular or sectorial shape composites. Increasing attention has been directed to mechanical problems of such composites. Recently, Feng et al.

**Table 1**
Simulation parameters in our numerical examples.

| Simulations parameters | Values | | | | | | |
|---|---|---|---|---|---|---|---|
| | Fig. 3 | Fig. 4 | Fig. 6 | Fig. 8 | Fig. 11 | Fig. 12 | Fig. 13 |
| Inner radius $r_0$ | 20 mm | 20 mm | 20 mm | 20 mm | Change | 200 mm | |
| Interface radius $r_1$ | 30 mm | change | change | change | $r_0$ + 5/3 mm | Change | $r_0$+5/3 mm |
| Outer radius $r_2$ | 40 mm | 40 mm | 40 mm | 40 mm | $r_0$ + 5 mm | Change | $r_0$ + 5/3 mm |
| Angle $\gamma$ | $2\pi/3$ | $2\pi/3$ | $\pi$ | Change | $\pi$ | | |
| Crack angle $\alpha$ | $\pi/5$ | $\pi/6$ | $\gamma/2$-$\pi/90$ | $\gamma/2$-$\pi/18$ | $\gamma/2$–2 mm/$r_1$ | | |
| Crack angle $\beta$ | $\pi/2$ | $\pi/2$ | $\gamma/2 + \pi/90$ | $\gamma/2 + \pi/18$ | $\gamma/2$ + 2 mm/$r_1$ | | |
| Stiffness ratio $\delta$ | 0.8 | 0.8 | Change | 0.5 | 1 | 1 | Change |
| **Simulations parameters** | **Values** | | | | | | |
| | Fig. 14 | Fig. 15 | Fig. 16 | Fig. 17 | Fig. 18 | Fig. 19 | |
| Inner radius $r_0$ | 10 mm | 10 mm | 20 mm | | | | |
| Interface radius $r_1$ | Change | 20 mm | Subgraph (a) 22 mm; (b) 30 mm; and (c) 38 mm | | | | |
| Outer radius $r_2$ | 30 mm | 30 mm | 40 mm | | | | |
| Angle $\gamma$ | $2\pi/3$ | $2\pi/3$ | $2\pi/3$ | | | | |
| Crack angle $\alpha$ | $\pi/6$ | Change $\beta - \alpha = \pi/6$ | $\gamma/2 - \pi/18$ | | $\gamma/2 - \pi/9$ | | |
| Crack angle $\beta$ | $\pi/2$ | | $\gamma/2 + \pi/18$ | | $\gamma/2 + \pi/9$ | | |
| Stiffness ratio $\delta$ | 0.8 | 0.8 | Change | | | | |