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Thermal stress intensity factor expressions for functionally graded cylinders with internal circumferential cracks using the weight function method

Iman Eshraghi *, Nasser Soltani

Intelligence Based Experimental Mechanics Center, Department of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran

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A B S T R A C T

In this paper, the weight function method is used to derive mathematical expressions in terms of the Gauss hypergeometric function for the mode-I thermal stress intensity factor of functionally graded cylinders with internal circumferential cracks. To determine the weight function coefficients, a unique function is fitted to reference stress intensity factors obtained from finite element analysis. Effects of the internal convection cooling coefficient and the material power law index are investigated, as well. It is shown that the thermal stress intensity factors predicted by the developed mathematical expression are in good agreement with those directly obtained from finite element analysis. Results of this study may be used in material selection, design optimization, safety assessment against fracture, and fatigue life evaluation of functionally graded cylinders.

1. Introduction

In many engineering applications, such as the power generation and chemical industries, structural components with cylindrical geometry subject to severe temperature gradients are often used. As a result, thermal stresses comparable in magnitude to the applied mechanical stresses exist in these structures. With the presence of small cracks, thermo-mechanical loads can result in the crack growth and final leakage or failure of the structure. Therefore, structural integrity assessment of these components is necessary when a flaw or crack exists. Circumferential cracks are often formed at the joint sections of pressure vessels and pipes. For example, cracks are often detected at a girth weld of a pipe due to the incomplete penetration of the weld material into the interface. Therefore, circumferential cracks in pipes and pressure vessels with homogeneous material properties have been the subject of research for many years (see, e.g., [1–5]).

Under extremely high temperature working conditions, functionally graded materials (FGMs) are capable of minimizing the effect of thermal stresses by a smooth change in their thermal and mechanical material properties along one (or more) direction (s). To date, researchers have investigated the behavior of functionally graded (FG) cylinders subjected to thermal and mechanical loads (see, e.g., [6–9]) and in some application cases thermal shocks have been identified as the source of the crack initiation in FG structures [10]. These cracks can grow under further thermal and/or mechanical loads and cause premature failure of the structure. Therefore, integrity assessment of FGMs with cracks subject to thermal and/or mechanical loads is necessary. Analytical and numerical approaches have been used to obtain stress intensity factor (SIF) and thermal SIF in FGMs. Martínez-Pañeda and Gallego [11] performed numerical and experimental studies to understand the quasi-static crack initiation and crack path growth in FG planar structures. Afsar and Anisuzzaman [12] analyzed pressurized thick-walled FG cylinders with two diametrically-opposed edge cracks. They considered the effect of residual thermal strains in their analysis to obtain SIFs. Recently, Guo and Noda [13] and Zhang et al. [14] studied the problem of thermal shock in FG structures with general thermo-mechanical material properties. They used the perturbation method along with the interaction energy integral method to analyze this problem.

The weight function method has been used to obtain SIF of homogeneous cylindrical structures with cracks. With the weight functions given for a crack configuration, it is possible to obtain SIFs for arbitrary loadings applied to the crack face [15,16]. This method has been widely used to develop closed form SIF expressions or tabulated SIF results for various crack geometries which are available in the codes of standard for the structural integrity assessment of cracked structures with homogeneous material properties [17].

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* Corresponding author at: North Karegar Ave., Jalal Ale Ahmad Blvd., Tehran, Tehran, Iran. Tel.: +98 9121433335.
E-mail address: ieshraghi@ut.ac.ir (I. Eshraghi).

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Nomenclature

a	crack depth
A	area of domain integral evaluation
A_{jk}	fitting parameters
Bi	Biot number, $Bi = R_i h_\infty / k_i$
E	elastic modulus
$\hat{E}$	modified plane stress/strain elastic modulus
${}_2F_1(p, b; c; z)$	Gauss hypergeometric function
h	convection coefficient
J	J-integral value
k	thermal conductivity
K_I	mode-I stress intensity factor
K_N	normalized mode-I stress intensity factor
K_r	reference stress intensity factor
$M_j \ (j = 1, 2, 3)$	weight function coefficients
q	a smooth function, which varies from unity to zero
$r, \ z$	radial and longitudinal directions in the cylindrical coordinate
R	radius of any material point inside the cylinder
$s(r, a)$	weight function
$S^U, \ S^L$	upper and lower surfaces of the crack in the integration domain
t	thickness of the cylinder
T	temperature distribution in the cylinder
$u_r, \ u_z$	radial and longitudinal components of the displacement field
W	strain energy density
$Y_j \ (j = 1, 2, 3)$	normalized reference stress intensity factors
$Z_j \ (j = 1, \ldots, 4)$	geometric parameters in the stress intensity factor expression
α	coefficient of thermal expansion
β	crack depth to cylinder wall thickness ratio $\beta = a/t$
$\gamma_j \ (j = 1, \ldots, 4)$	parameters dependent on λ
δ_{AB}	Kronecker delta
$\eta_j \ (j = 1, \ldots, 4)$	constants dependent on thermal and mechanical boundary conditions
λ	power law index for functionally graded material
ν	Poisson's ratio
σ_0	reference uniform stress applied to the crack faces
$\sigma_{zz}(r)$	axial stress component
χ	generic material property

Subscripts

$()_i$	inner surface of the cylinder
$()_o$	outer surface of the cylinder
$()_{tip}$	crack tip location
$()_\infty$	fluid inside the cylinder

Analytical approaches for the analysis of fracture problems in FGMs [18] are limited to specific geometries of the cracked body, crack configuration, and loading conditions. Therefore, to obtain SIFs for complex geometries and load cases, numerical approaches such as the boundary domain element method [19], standard FE method with the energy integral methods [20,21] or the extended finite element method [22] have been used. Energy-based FE methods can accurately determine SIFs for cracked FG structures; however, extensive FE analysis is required for every individual crack geometry and its loading condition. By combining the weight function method and the energy-based FE method it is possible to reduce the number of FE analyses required for a given crack geometry under different loading conditions in FG structures [23,24]. There are few works in the literature which have applied the weight function method to FG cracked structures. Bahr et al. [25] applied the weight function method to determine SIFs in FG structures with residual stresses. Shi et al. [26] proposed basic weight function equations for two-dimensional FG cracked structures using Betti's reciprocal theorem. Seifi [27] calculated residual stresses for autofrettaged FG cylinders and obtained SIFs at the deepest and surface points of a semi-elliptical axial crack using the weight function approach. Recently, the authors have applied the weight function method to FG cylinders with internal circumferential cracks and have shown the accuracy of the method in predicting the SIFs [28]. A modified form of the J-integral for axisymmetric FG cylinders under applied mechanical loads was used in [28] to obtain reference SIFs from the FE analysis results.

In this work, the weight functions proposed by Glinka and Shen [29] are used to calculate thermal SIFs of circumferential cracks in FG cylinders. The FG cylinder is under internal and external pressure loads as well as internal convection cooling. Thermo-mechanical material properties of the FG cylinder vary through the wall thickness according to a power law equation. The steady state temperature distribution and the resulting thermal stresses in the cylinder are discussed. Using the weight function expression and the axial component of the thermal stress, analytical expressions in terms of the Gauss hypergeometric function are derived for the mode-I thermal SIF of the FG cracked cylinder. The weight function coefficients are determined using reference SIF results obtained from the FE analyses for three reference crack surface loads. The modified form of the J-integral in domain form for axisymmetric FG structures, presented in [28], is further extended to include the effect of thermal loading and is implemented in the post-processing step of the FE analysis to calculate the reference SIFs. Furthermore, a unique fitting function is introduced to interpolate the reference SIFs for all three reference load cases and for arbitrary values of the crack depth to cylinder thickness ratio and the material power law index not covered in the FE modeling matrix. Coefficients of the proposed curve fitting equation are determined using the obtained reference SIFs from the FE modeling. Comparison of the thermal SIF results predicted by the weight function method with those of the direct FE analysis results is presented. Effects of the Biot number, the crack depth to cylinder thickness ratio, and the FG power law index on the thermal SIF results are discussed, as well.

2. Thermal stresses for FG cylinders subject to internal cooling

An infinitely long thick-walled cylinder with inner radius R_i, outer radius R_o, wall thickness $t = R_o - R_i$, and a complete internal circumferential crack of depth a is shown in Fig. 1(a). The cylinder is subjected to internal cooling with a convection heat transfer coefficient of h_∞ and internal fluid temperature T_∞. The cylinder outer surface temperature is kept constant at T_o and internal pressure p_i and external pressure p_o are applied to the inner and the outer surfaces of the cylinder, respectively. Since the cylinder is assumed to be very long, plane strain conditions exist in the cylinder. The cylinder is made of a FG material with thermal and mechanical properties that vary through the cylinder wall thickness according to the following equation

$$\chi = \chi_i \left(\frac{r}{R_i} \right)^\lambda \tag{1}$$