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Tension analysis of infinite solid circular cylinders with arbitrary located axisymmetric cracks

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ABSTRACT

This paper deals with the mixed mode crack problem in a long circular cylinder of elastic material. First, the solution of axisymmetric Volterra climb and glide dislocations in an infinite circular cylinder is obtained by making a suitable representation of the biharmonic stress function. Next, the distributed dislocation technique is used to formulate integral equations for a system of coaxial axisymmetric cracks, including penny-shaped, annular and circumferential cracks. The cylinder is under the action of two distributed self-equilibrating shear tractions on the curved surface of cylinder. These equations are solved numerically to obtain the dislocation density on the surfaces of the cracks. The dislocation densities are employed to determine stress intensity factors for axisymmetric cracks. Several examples are presented to study the crack type/location on the ensuing stress intensity factors at tips of cracks and also the interaction between the cracks.

1. Introduction

Because of massive applications of shafts as the rotation members which are used to transmit torque, power and motion between various elements, the stress analysis of them is of special interest. The shafts are generally made in the cylindrical shapes which can be solid or hollow and are subjected to different types of loadings. The circular cylinders as a subgroup of the shafts are generally vulnerable to cracking. These cracks may initiate during the manufacturing process or in the regions subjected to steep stress gradient in the course of service life of a cylinder. Multiple cracks with any shape and direction may exist in a circular cylinder making the analytical stress analysis of it intractable. Therefore, only restricted geometries and configurations of cracks may be tackled without resorting to approximation methods. The existence of the axisymmetric cracks as the cracks with special geometries, causes the mixed mode crack problem with three modes of fracture to be transformed to a problem with only two modes. Although the cylinder under three mixed modes of fractures is more realistic from practical point of view but the cylinder subjected to two mixed modes is prerequisite for it. Therefore in this study we confine ourselves to the axisymmetric cracks.

The effect of the axisymmetric cracks such as penny-shaped, annular cracks as well as circumferential edge cracks on distribution of stresses in geometrically different domains has been studied frequently [1–22]. These domains include the infinite domains [1–5] as well as the solid [6–22] and hollow cylinders [24,27]. Because of the space limitation, we focus our review only on the solid cylinders.

The displacement and stress field can be stated in terms of a biharmonic stress function which is an efficient means to solve the axisymmetric crack problems [6–8]. These problems were reduced to the solution of a Fredholm integral equation of the second kind. In what follows, we review some of the above-mentioned studies in detail.

Distribution of stress in an infinite circular cylinder of elastic material with an eccentric symmetric penny-shaped crack under pressure to the inner surfaces of the crack was obtained by Sneddon and Tait [6]. It was assumed that the cylindrical surface was free of shear stress and the radial component of the displacement vector vanishes on this surface. The governing partial differential equation and also the displacement and stress components were given in terms of biharmonic stress function. By making an appropriate type of biharmonic function in the integral form, the problem was reduced to the solution of a Fredholm integral equation of the second kind. In the particular case when the crack was opened up by a constant pressure, the basic integral equation was solved by an iterative procedure to give expressions for the quantities such as the critical pressure and the stress intensity factor of the crack tip. The above problem for a cylinder with a stress-free lateral surface was re-examined by Sneddon and Welch [7].

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Using a similar method of solution, an isotropic long circular cylinder with an infinite number of parallel identical penny-shaped cracks was studied by Watanabe and Atsumi [8]. The cracks were arrayed periodically in the direction of the cylinder axis and were opened by the equal inner pressure on each of the surfaces of the cracks. The problem was modeled by a system of short circular cylinders with a concentric crack in each of them. An iterative method was used to solve the Fredholm integral equation of the problem. For each crack, the quantities such as stress intensity factor of the crack tip, deformation of the crack surface, strain energy and critical pressure were calculated.

The basic axisymmetric crack problem can be also superposed to some simple problems such as the studies [9–11]. This is another efficient method to analyze the related studies. In this regard, Kaman and Gecit [9] analyzed a cracked semi-infinite isotropic cylinder as well as a finite isotropic cylinder with no crack. One end of the cylinders was fixed while the other end was under an axial tension. The problem of a cracked semi-infinite isotropic cylinder was superposed to two problems. Namely, an infinite cylinder subjected to remote uniformly distributed tensile load and also an infinite cylinder with a penny-shaped rigid inclusion and two identical penny-shaped cracks located at the same distance above and below the inclusion. Extension of the radius of the inclusion divides the infinite cylinder to two semi-infinite cylinders, each with one penny-shaped crack. The superposed problems were analyzed by solving Navier's equations with the Fourier and Hankel transforms. Formulation of the problem was reduced to a system of three singular integral equations which were solved numerically. Similarly, they reexamined the problem of the finite cylinder for that of containing a penny-shaped crack with one fixed end while the other end was under uniform tension [10]. Similarly, Toygar and Gecit [11] investigated the axisymmetric crack problem of an infinite isotropic cylinder with two identical annular rigid inclusions of negligible thickness. The cylinder was also weakened by an annular crack located exactly between the inclusions and was under a remote axisymmetric tension. The problem reduced to three singular integral equations in terms of the derivative of the crack surface displacement and also the stress jumps on the rigid inclusions. Numerical solution of the above integral equations in conjunction with the single-valuedness condition of the displacements out of the crack and the equilibrium equations along the inclusions leads to stress intensity factor of the crack tip.

The axisymmetric cracks problems we wish to address here are those which were solved by stating the axisymmetric displacements and stresses in terms of two potential functions [12–14]. Reviews of these papers are given in the following.

The analysis of the singular stresses distribution in a transversely isotropic infinite cylinder with a circumferential edge crack subjected to a uniform axial tension was studied by Atsumi and Shindo [12]. The axisymmetric displacements and stresses satisfying the equilibrium equations were stated in terms of two harmonic potential functions. Using the Hankel transform, the problem was reduced to a Cauchy singular integral equation of the first kind which was solved numerically. The singular stresses are expressed in closed form and the stress intensity factor of the crack tip was obtained.

The analysis of the stress distribution in an infinite circular cylinder of elastic material with a concentric penny-shaped crack under a uniform shearing stress was conducted by Lee [13]. The cylindrical lateral surface was stress-free. Two solution forms were proposed in the paper. In the first one, the displacements and stress components were given in terms of a biharmonic and a harmonic function. The biharmonic and harmonic functions were appeared as integrals with integrands involving the modified Bessel functions of the first kind. In the second one, the displacements and stress were stated in terms of two different harmonic functions with integrals containing Bessel functions of the first kind. By choosing the proper solution form, the boundary conditions of the problem were applied and the problem was reduced to the solution of a pair of Fredholm integral equations of the second kind. Those were solved numerically, and the stress intensity factor of the crack tip was evaluated. Similarly a cylinder with a circumferential edge crack subjected to a uniform shearing stress was accomplished by Lee [14]. The similar method of solution explained in the reference Lee [13] was used.

Among the different solution methods of the axisymmetric crack problems [1–22], some non-routine methods are reviewed in the following:

Using the body force method, the problem of an isotropic long circular cylinder with an infinite number of parallel identical circumferential edge cracks was investigated by Nisitani and Noda [15]. The cylinder was subjected to tension. By superposing the stress fields of a periodic array of ring forces, the boundary conditions of the problem were satisfied. The body force densities were determined by solving a system of algebraic equations and using those, the stress intensity factors of the crack tips were calculated.

Qizhi [16] studied the problem of a cylinder with a concentric penny-shaped crack under various loading conditions. In that paper, the loading were as follows:

(1) A constant pressure in a circular area on the crack surfaces. (2) Point forces at the center of crack. (3) A torsional loading with shear stress applied linearly to the crack faces. Based on the crack-plane stress field method, a suitable stress component on the crack plane of a penny-shaped crack located in an infinite space was used to solve the main problems. That is, a modification factor was multiplied to the singular term of above-mentioned stress component and using the balance condition on the crack extension plane this factor was calculated. Finally using the modified stress field, the SIF of a cylinder with a concentric penny-shaped crack was derived for all aforementioned loadings.

Danyluk et al. [17] studied the problem of a transversely isotropic cylinder made of elastic perfectly-plastic material weakened by a concentric penny-shaped crack. The crack surfaces were prescribed by a constant pressure. The curved surface of the cylinder was free of stress. For linear elastic materials, the Dugdale hypothesis was used to remove the singularity of the normal stress at the leading edge of the crack. The problem was reduced to a Fredholm integral equation of the second kind in terms of the width of the plastic zone which was solved numerically.

As we know within the framework of the fracture mechanics, two-dimensional conservation law, have two components including J integral and G^* integral. The J integral deals with crack extension energy release rate while the G^* integral deals with the crack mouth widening energy release rate which are calculated for the same crack problem. These integrals can be used for evaluation of the Mode I stress intensity factor of the three-dimensional crack. Xie et al. [18] treated the problems of homogenous and composite cylinders with a penny-shaped crack using the G^* integral. Cylinders were subjected to two kinds of loadings that is, tension or lateral loading. By application of the principle of the virtual work, the relationships between three-dimensional G^* integral and stress intensity factor of the crack tip were derived.

The dislocation distributed technique is a capable tool for analyzing the multiple cracks in the domains of arbitrary form. The only valuable dislocation solution for the axisymmetric crack problems is the paper presented by Asadi et al. [19]. First, the solution of axisymmetric Volterra climb and glide dislocations in an infinite domain was given. Using the Papkovich–Neuber potentials, the