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Interaction between multiple piezoelectric inclusions and a crack in a non-piezoelectric elastic matrix

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ABSTRACT

In this paper, a general method is presented for evaluating the interaction between multiple piezoelectric inclusions and a nearby crack in a non-piezoelectric elastic matrix. The elastic matrix is subjected to a uniform far field in-plane tension and all inclusions are subjected to an out-of-plane uniform electric field. The crack in the elastic matrix is treated as a continuous distribution of edge dislocations, and then the solution of a unit edge dislocation interacting with multiple piezoelectric inclusions in an elastic medium is derived as the Green function. The problem is formulated into a set of singular integral equations which are solved by a numerical method, and the stress intensity factors (SIFs) at the crack are obtained in terms of the dislocation density functions evaluated from the singular integral equations. Numerical examples are given for a few typical arrays of piezoelectric inclusions with various material properties and geometric parameters. The results indicate that the applied uniform electric-field plays an important role in the interaction between multiple piezoelectric inclusions and the matrix crack., Moreover, it is found that the influences of 'softer' piezoelectric inclusions on the SIFs are quite different from that of 'harder' piezoelectric inclusions, and the SIFs at the crack tip are greatly affected by the geometry and array of piezoelectric inclusions.

1. Introduction

Fiber-reinforced composites are widely used in engineering fields such as aerospace structures, submersible vehicles and offshore structures due to their relatively high strength and stiffness and low density. However fibers (inclusions) introduced as strengthening material phases destroy the homogeneity of the matrix and lead to local stress concentration around the inclusions. Meanwhile, unavoidable defects such as micro-cracks in the matrix may worsen the performance of the composites. Thus it is necessary to study the interaction between inclusions and matrix cracks for developing high performance composites. The inclusion-crack-matrix interaction problems have received a considerable attention recently. Earlier researchers, such as Tamate [1], Atkinson [2], Erdogan et al. [3], and Hsu and Shivakumar [4] analytically investigated the problem of a circular inclusion interacting with a crack in elastic matrix. Then Erdogan and Gupta [5], Bhargava and Bhargava [6], Isida and Noguchi [7], Wu and Chen [8], Kim and Sudak [9], Liu and Ru [10] studied more complicated cases with multiple cracks, an elliptical inclusion or a three-phase circular inclusion. Recently, Li et al. [11] studied a screw dislocation interacting with a nanoscale circular inclusion and a model III crack by the complex potential function method. Numerical approaches such as finite element method (FEM) [12–14] and boundary element method (BEM) [15–17] were also developed to deal with more general situations with increasing technical complexity. In addition, the method of distributed dislocation is also an effective tool for a crack contained in a matrix [18–21], and alternatively a crack can also be simulated by body force method [22].

Recently there has been growing interest in 'smart materials' due to their intrinsic electromechanical coupling behavior. In general, piezoelectric fibers, based on ferroelectric crystals such as lead zirconate titanate (PZT) and barium titanate (BaTiO3), are widely employed as electromechanical sensors, transducers and actuators [23,24] in smart composites. Because of practical relevance of piezoelectric composites for engineering applications, electro-elastic analysis of such materials has become one of the most popular research topics. For example, Tan and Tong [25] proposed two micro-electromechanics models (a rectangle model and a rectangle-cylinder model) to predict elastic, piezoelectric and dielectric

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constants of piezoelectric fiber-reinforced composite (PFRC) under single load or multiple loads. Yang and Gao [26] studied electro-elastic fields in an infinite matrix with N coated piezoelectric inclusions based on the complex variable method. Dunn and Wienecke [27] studied electro-elastic field inside and around inclusions and inhomogeneities embedded in a transversely isotropic piezoelectric solids using Eshelby's approach. It is noticed that most existing research works are restricted to the problems of piezoelectric inclusions in the absence of any cracks in the matrix. Practically, however, matrix cracking may also exist in piezoelectric composites during manufacturing process or under applied tensile mechanical stress or electrical loading. Little work has been done on piezoelectric inclusions interacting with a matrix crack, except Xiao and Bai [28] who provided a solution for a single piezoelectric inclusion interacting with a matrix crack. In many practical problems, multiple piezoelectric fibers are used as sensors or actuators in 'smart materials'. Although the single inclusion model is adequate for sparsely arrayed fibers, it is certainly inadequate densely arrayed fibers. The interaction between multiple fibers and a matrix crack represents a significant research topic of practical relevance.

The purpose of this work is to present a solution to the problem of closely arrayed multiple piezoelectric inclusions interacting with a near-by crack in a non-piezoeletric elastic matrix. In Section 2, a procedure is used to decompose the problem into two sub-problems. Section 3 solves the two sub-problems respectively and then provides a solution of the original problem. Some numerical results are given in Section 4 to show the influence of the applied electric field, array of inclusions, geometric parameters and material properties on the SIFs at the matrix crack. Main conclusions are summarized in Section 5.

2. Problem description

In a rectangular coordinate system x_j ($j = 1,2,3$), we consider an infinite isotropic elastic matrix containing N parallel cylindrical piezoelectric inclusions (fibers) aligned along the x_3 direction. A through-x_3 direction matrix crack of length 2c along the x_1-axis locates along the x_1-axis and is centered at the origin of the x_1–x_2 plane, as shown in Fig. 1. It is assumed that the elastic matrix is isotropic, while the piezoelectric inclusions are transversely isotropic and polarized along the symmetry axis x_3. The matrix is subjected to far-field mechanical stresses and the inclusions are loaded by a uniform electric field E_3^∞ in the x_3 direction. Additionally, all inclusions are assumed to be perfectly bounded to the matrix. The cross-section of the system is shown in Fig. 1, where the regions occupied by the matrix and the inclusions are denoted by subscripts 'm' and 'f', respectively, the shear modulus and the Poisson's ratio of the elastic matrix and the inclusions are μ_m and μ_f, and v_m and v_f, respectively, and R_r ($r = 1, 2, 3, \ldots, N$) represents the radius of the rth inclusion l_r.

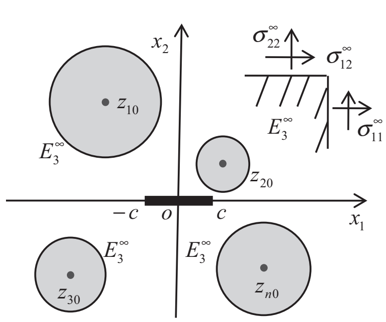

Fig. 1. The interaction between multiple circular piezoelectric inclusions and a crack in an infinite elastic matrix.

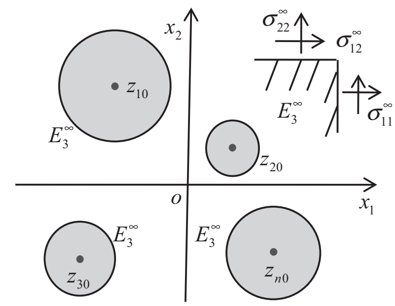

Fig. 2. Sub-problem I.

As the matrix is pure elastic material, there is no mechanical-electric coupling inside the matrix. By employing the superposition principle of elasticity [29], the solution of the present problem can be obtained as the sum of two sub-problems, as shown in Fig. 1. The sub-problem I shown in Fig. 2 is the piezoelectric inclusions embedded in the elastic matrix without the matrix crack. For the sub-problem II shown in Fig. 3, the only external loads are the crack surface tractions which are equal in magnitude and opposite in sign to the stresses obtained in the sub-problem I along the crack faces. The superposition of sub-problem I and sub-problem II thus gives the solution of the original problem.

3. Solution procedure

The sub-problem I has been solved in ref. [30]. In the matrix (an infinite plane with N circular holes), the complex potentials can be written in the form of power series

$$\begin{aligned}
\phi_m^I(z) &= (B_1 + iC_1)z + \sum_{r=1}^{N}\sum_{k=1}^{\infty}\alpha_{-k}^{(r)}\left(\frac{R}{z - z_{r0}}\right)^k \\
\psi_m^I(z) &= (B_2 + iC_2)z + \sum_{r=1}^{N}\sum_{k=1}^{\infty}\beta_{-k}^{(r)}\left(\frac{R}{z - z_{r0}}\right)^k,
\end{aligned} \tag{1}$$

where $\alpha_{-k}^{(r)}$ and $\beta_{-k}^{(r)}$ are unknown coefficients, z_{r0} is the centre of the r-th inclusion, R stands for a reference length which may be defined as $R = \min\{R_1, R_2, R_3, \ldots, R_N\}$, and B_i, C_i ($i = 1, 2$) are related to the applied uniform stresses at infinity:

$$B_1 = (\sigma_{11}^\infty + \sigma_{22}^\infty)/4, C_1 = 0, B_2 = (\sigma_{22}^\infty - \sigma_{11}^\infty)/2, C_2 = \sigma_{12}^\infty. \tag{2}$$

On the other hand, the complex potentials inside the inclusions l_p can be expressed as

$$\begin{aligned}
\phi_f^I(z) &= \sum_{k=0}^{\infty}\hat{\alpha}_k^{(p)}\left(\frac{z - z_{p0}}{R}\right)^k, \\
\psi_f^I(z) &= \sum_{k=0}^{\infty}\hat{\beta}_k^{(p)}\left(\frac{z - z_{p0}}{R}\right)^k \quad (p = 1, 2, \ldots, N)
\end{aligned} \tag{3}$$

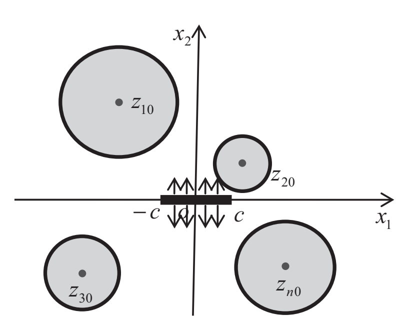

Fig. 3. Sub-problem II.