ARTICLE IN PRESS

Energy xxx (2015) 1–11

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

# Entropy generation analysis of laminar forced convection through uniformly heated helical coils considering effects of high length and heat flux and temperature dependence of thermophysical properties

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ARTICLE INFO

*Article history:*
Received 18 September 2014
Received in revised form
10 January 2015
Accepted 13 January 2015
Available online xxx

*Keywords:*
Entropy generation
Helical coils
Temperature dependent thermophysical properties
Optimization

ABSTRACT

In this study, combined effects of length and heat flux of the coil as well as the effects of temperature dependence of thermophysical properties on entropy generation rates and optimal operation of uniformly heated helical coils with laminar forced convection have been analyzed analytically. For these purposes, comprehensive analytical formulas, which could be used for any duct shape and flow regime, are derived for thermal, frictional, and total entropy generation rates, and the effects of involved parameters on the entropy generation rates are examined for laminar forced flow of water through uniformly heated helical coils. Then, using the minimal entropy generation principle, the inlet Reynolds number is optimized for various values of the involved parameters, and some correlations are proposed for optimal values of this parameter which extend and modify the existing correlations of water. It is found that the entropy generation rates are highly dependent on the combined effects of length and heat flux of the coil, introduced by the parameter $\eta_C$, and temperature dependence of thermophysical properties, such that all of them noticeably augment with increase in $\eta_C$ and the inlet temperature.



## 1. Introduction

Heat exchangers have been widely used in many industrial applications, such as food processing, piping systems, chemical reactors, and electronics cooling due to the critical need of these applications for heat dissipation. As any other industrial device, many engineering efforts have been put into increasing their performance. The methods which have been proposed to enhance the efficiency of heat exchangers can be classified into three main groups of active, passive, and compound remedies [1]. Active methods make use of an external supply of power to increase the heat transfer of heat exchangers, such as vibrating the heat transfer surfaces or the fluid. On the other hand, passive methods bring about the enhancement in heat transfer by making geometrical modifications to the flow passage, like curving or twisting the flow passages. Compound methods make use of a combination of the active and passive methods, such as using rough surfaces with fluid vibration. One important point that should be noted here is that, as discussed before, passive methods do not need any external power input, which gives them a great advantage over the active methods in terms of overall efficiency and reliability of the whole system.

Among the mentioned passive methods, the technique of coiling the flow passages has received great attention due to its simplicity, flexibility, and huge effectiveness, not to mention its great potential for constructing compact heat exchangers which is of major concern in today's adopted trend of miniaturization. The curvature of coiled pipes induces a secondary flow motion which causes their heat transfer and pressure drop to be greater than those in straight pipes. Accordingly, heat transfer and flow in these pipes have been the focuses of many investigations in the past decades [2–5].

As any other heat exchanger, the critical design challenge of curved ducts is the increment of their pressure drop with any attempt for enhancing their heat transfer rate [6]. The increase in the heat transfer rate leads to decrement in thermal entropy generation rate, and the pressure drop increment causes augmentation of frictional entropy generation rate. Thus, for the curved passages, the existence of an optimal condition with the minimal total entropy generation rate is obvious. So far, many researchers have incorporated the systematic methodology described by Bejan [7] and have proposed thermodynamically optimized designs of

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http://dx.doi.org/10.1016/j.energy.2015.01.041
0360-5442/

## Nomenclature

| | |
|---|---|
| $A$ | cross-sectional area of the flow passage ($m^2$) |
| $a$ | curvature radius of the coil ($m$) |
| $b$ | coil pitch ($m$) |
| $C_p$ | constant pressure specific heat (J/(kg K)) |
| $C_p^*$ | dimensionless constant pressure specific heat, $C_p^* = C_p/C_{pin}$ |
| $D_h$ | hydraulic diameter of the flow passage, $D_h = \frac{4A}{p}$ |
| $Dn$ | Dean number, $Dn = Re\delta^{1/2}$ |
| $e$ | specific enthalpy (J/kg) |
| $f$ | fanning friction factor, $f = -\frac{D_h}{2\rho V^2}\frac{dP}{dt}$ |
| $He$ | helical number, $He = Dn/[1+\lambda^2]^{1/2}$ |
| $h$ | heat transfer coefficient of the tube side of the flow passage (W/(m$^2$ K)) |
| $k$ | thermal conductivity coefficient (W/(m K)) |
| $k^*$ | dimensionless thermal conductivity, $k^* = k/k_{in}$ |
| $L$ | length ($m$) |
| $\dot{m}$ | mass flow rate (kg/s) |
| $N_s$ | total entropy generation number |
| $(N_s)_P$ | frictional entropy generation number |
| $(N_s)_T$ | thermal entropy generation number |
| $Nu$ | Nusselt number, $Nu = \overline{h}D_h/k$ |
| $P$ | pressure (Pa) |
| $p$ | perimeter of the flow passage (m) |
| $Pr$ | Prandtl number, $Pr = \mu C_p/k$ |
| $q'$ | heat transfer rate per unit length of the flow passage (W/m), $q' = q''p$ |
| $q''$ | heat flux of the flow passage (W/m$^2$) |
| $Re$ | Reynolds number, $Re = \rho V(2r_0)/\mu$ |
| $r_0$ | inner radius of the coiled tube ($m$) |
| $s$ | specific entropy generation (J/(kg K)) |
| $\dot{S}'_{gen}$ | entropy generation rate per unit length of the flow passage (W/(m K)) |
| $\dot{S}_{gen}$ | entropy generation rate of the whole coil (W/K) |
| $T$ | fluid temperature (K) |
| $T^*$ | dimensionless temperature, $T^* = T/T_{in}$ |
| $t$ | location coordinate along the curve of the flow passage ($m$) |
| $t^*$ | dimensionless location coordinate along the curve of the flow passage, $t^* = t/L_{fp}$ |
| $V$ | average fluid velocity in the flow passage (m/s) |
| $v$ | specific volume (m$^3$/kg) |
| $X$ | location coordinate along the coil axis ($m$) |

*Greek letters*

| | |
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| $\Delta$ | difference operator |
| $\delta$ | curvature ratio, $\delta = r_0/a$ |
| $\eta_C$ | combined length and heat flux characteristic of the flow passage, $\eta_C = \frac{q''pL_{fp}}{\dot{m}\overline{C_p}T_{out}} = \frac{q''pL_{coil}\sqrt{1+\lambda^{-2}}}{\dot{m}\overline{C_p}T_{out}} = 1 - \frac{T_{in}}{T_{out}}$ |
| $\xi_1$ | first inlet duty parameter, $\xi_1 = \frac{k_{in}T_{in}}{D_h q''}$ |
| $\xi_2$ | second inlet duty parameter, $\xi_2 = \frac{32q''\dot{m}^2\rho_{in}^2D_h^3}{p^2\mu_{in}^5}$ |
| $\lambda$ | dimensionless pitch, $\lambda = b/(2\pi a)$ |
| $\mu$ | molecular viscosity (Pa s) |
| $\mu^*$ | dimensionless molecular viscosity, $\mu^* = \mu/\mu_{in}$ |
| $\rho$ | density (kg/m$^3$) |
| $\rho^*$ | dimensionless density, $\rho^* = \rho/\rho_{in}$ |

*Subscripts*

| | |
|---|---|
| $b$ | property of the bulk of the fluid |
| $coil$ | associated with the coil |
| $crit$ | critical |
| $fp$ | flow passage |
| $in$ | inlet ($t^* = 0$) |
| $out$ | outlet ($t^* = 1$) |
| $w$ | associated with the wall of the flow passage |

*Superscripts*

| | |
|---|---|
| $-$ | temperature-averaged between $T_{in}$ and $T_{out}$ |

curved tubes. Ko and Ting [8] studied the entropy generation of fully developed laminar forced convection of air through a helically coiled tube under constant wall heat flux and calculated optimal states in some operational conditions. Ko and Ting [9] extended their study to encompass more general configurations for the flow and calculated the optimal values of Reynolds number for water and air. Ko [10] considered fully developed laminar forced convection of water and air through uniformly heated helical coils and optimized the curvature ratio of the coil. Ko [11] calculated the optimal values of mass flow rate for fully developed laminar forced convection of water and air through helically coiled tubes under constant wall heat flux. Shokouhmand and Salimpour [12] considered the same flow regime of fully developed laminar forced convection and analyzed the entropy generation and optimal Reynolds number of helical coils under constant wall temperature for water and air. Shokouhmand and Salimpour [13] extended their previous work and calculated optimal values of Reynolds number for more general flow configurations. Satapathy [14] studied thermodynamic irreversibilities in a coiled tube heat exchanger under constant wall heat flux for both laminar and turbulent flow regimes and found the optimum coil to tube diameter ratio. Bahiraei et al. [15] investigated the thermodynamic potential for improvement of laminar steady state fully developed forced convection of water through helical coils under constant wall temperature, based on the concept of avoidable and unavoidable exergy destruction, and showed considerable potential of thermodynamic optimization of helical coils. Amani and Nobari [16] analyzed and optimized the entropy generation of incompressible viscous flow in the entrance region of curved pipes subjected to constant wall temperature, numerically. The common feature of all of these studies is that they have assumed constant thermophysical properties of the working fluid along the flow passage in their analyses. Moreover, an implicit and inherent feature of all the studies performed on helical coils with constant wall heat flux, although not mentioned in them, is that they are only valid for coils with a very small temperature difference between their inlet and outlet. More exactly speaking, as is shown through this article, these studies are exact for just one section of a practical coil.

In the present study, the effects of temperature dependence of thermophysical properties of the working fluid as well as the impacts of high length and heat flux of the flow passage on entropy generation rates and optimal Reynolds number of fully developed laminar forced convection through uniformly heated helical coils is analyzed. For this purpose, relevant entropy generation formulas are derived, effects of high temperature difference between the inlet and the outlet of the coil, originated from the large length or high heat flux of the coil, as well as temperature dependence of thermophysical properties on entropy generation rates are examined, and optimal designs are obtained via the minimal entropy generation principle.