Theoretical and Applied Fracture Mechanics 76 (2015) 50–59

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Evaluation of the singularity exponents and characteristic angular functions for piezoelectric V-notches under in plane and out of plane conditions

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ARTICLE INFO

Article history:
Available online 12 January 2015

Keywords:
Piezoelectric field
V-notch
Singular stress field
Asymptotic analysis

ABSTRACT

Based on the assumption that the physical fields close to the V-notch vertex are expressed by the series asymptotic expansions, the evaluation of singularity exponents for piezoelectric V-notches is turned into solving the characteristic values of ordinary differential equations under given boundary conditions, which are a set of equations with variable coefficients and solved by the interpolating matrix method developed by part of the authors before. The singularity analysis for V-notches under in plane and out of plane conditions is taken into account. Numerical results show that all the singularity exponents and the characteristic angular functions can be evaluated synchronously without the need of solving the transcendental equations iteratively. The present method is not only suitable for the singularity analysis of V-notches, but also for cracks and interface ends, without encountering the problem of ill-posed equations when the complex potential function method for the singularity analysis of V-notches is degenerated to analyze the singularity of cracks.

1. Introduction

The piezoelectric materials have been widely used in modern technical areas such as smart structures, mechatronics and micro-system technologies, where they are used as actuators or sensors. The V-notch configurations are often encountered in piezoelectric structures, such as the interfacial end of piezoelectric laminated plates [1–3], and the interfacial crack between the metal and piezoelectric material [4,5]. Due to the sharp changing of the geometry and/or the material property, the singular electro-mechanical field will appear at the apex of a piezoelectric V-notch. The geometrical and material singularities submitted to electrical and mechanical fields will cause the crack initiation and fatigue crack propagation [6,7]. The safety of the piezoelectric engineering structures is highly affected by the singular stress field close to the vertex of the V-notch [8,9]. The failure assessment requires efficient analytical and numerical techniques in order to determine this stress field close to the V-notched piezoelectric structures.

As in the case of V-notches, the singularity at crack tips and interfacial ends are paid extensive attentions by the researchers. The interfacial crack in piezoelectric materials was investigated using the complex variable function method [10,11]. The influence of the electro-elastic interactions on the stress intensity factor of an interfacial crack was studied by Narita and Shindo [12]. The singularity analysis for edge-cracked piezoelectric material was confirmed by the Hamiltonian transformation [13]. The mechanical strain energy release rate for the crack in piezoelectric materials was given out by the Mellin transform method [14,15].

In comparison with classical fracture mechanics, the singular stress field at the tip of a V-notch is more complicated to determine. Nevertheless, the electro-elastic singularity analysis for the crack offers a fruitful reference way to investigate the singularity for the V-notch case. A semi-analytical technique to analyze 2-D cracks and V-notches existing in piezoelectric composites was proposed based on the scaled boundary finite element method [16]. The stress intensity factor of an interfacial corner between piezoelectric bi-materials was derived by the Stroh formalism [17,18]. The asymptotic behavior at piezoelectric material interface corner configurations was described by combing the eigenfunction expansions with the regular finite element method [19]. The singular electro-elastic field of the piezoelectric material near the V-notch tip was studied by the boundary element method [20]. The singular characteristic solutions of a piezoelectric V-notch were obtained by the finite element method [21,22,1]. The singular

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http://dx.doi.org/10.1016/j.tafmec.2015.01.001
0167-8442/

Nomenclature

$or\theta$	polar coordinate system
oxy, $ox'y'$	Cartesian coordinate systems
β	angle of polarization direction
θ_1, θ_2	angles of two radial edges of V-notch
$\sigma_x, \sigma_y, \tau_{xy}$	plane stress components in Cartesian coordinate system
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$	plane strain components in Cartesian coordinate system
D_x, D_y	electric displacement components in Cartesian coordinate system
E_x, E_y	electric field strength components in Cartesian coordinate system
$\sigma_r, \sigma_\theta, \tau_{r\theta}$	plane stress components in polar coordinate system
$\varepsilon_r, \varepsilon_\theta, \gamma_{r\theta}$	plane strain components in polar coordinate system
D_r, D_θ	electric displacement components in polar coordinate system
E_r, E_θ	electric field strength components in polar coordinate system
$C_{11}, C_{13}, C_{33}, C_{44}$	elastic modulus
e_{13}, e_{15}	piezoelectric constants
k_{11}, k_{33}	dielectric constants
u_r, u_θ	plane displacement components in polar coordinate system
ϕ	electric potential
A_k	amplitude coefficient in asymptotic expansions
λ_k	singularity exponent
N	number of truncated series item
$\tilde{u}_{ik}(\lambda_k, \theta)(i = r, \theta)$	plane displacement characteristic angular functions
$\tilde{\phi}_k(\lambda_k, \theta)$	electrical potential characteristic angular functions
σ_{xz}, σ_{yz}	shear stresses in Cartesian coordinate system
γ_{xz}, γ_{yz}	shear strains in Cartesian coordinate system
$\sigma_{rz}, \sigma_{\theta z}$	shear stresses in polar coordinate system
$\gamma_{rz}, \gamma_{\theta z}$	shear strains in polar coordinate system
w	z-direction displacement component
$\tilde{w}_k(\lambda_k, \theta)$	z-direction displacement characteristic angular functions

behavior of electro-elastic fields at the corner of wedges and junctions were studied by using Lekhnitskii's complex potential functions [23]. The expressions of singularity orders (singularity exponents) and generalized stress intensity factors of out of plane case were deduced by the Mellin transformation [24]. The failure behavior of conductive deep notches in piezoelectric ceramics was studied through the comparison of failure criterion with experimental verification [25]. The explicit forms of singular electro-elastic stress field in a piezoelectric material containing a V-notch under the generalized plane strain and the out of plane shear loading were proposed using the complex potential function method associated with eigenfunction expansion method [26,27].

Some of the methods mentioned above, allowing to study the singular behavior close to V-notches tips, need to solve the transcendental equations by iterative methods, and some of them cannot be degenerated to analyze the crack problems. In addition, at most only the first two orders of singularity can be obtained by the experimental verification. Herein, a novel numerical method is proposed for the singularity analysis of the piezoelectric material containing a V-notch. Based on the asymptotic expansion of the physical field with respect to the radial distance from the vertex, the governing equations and the mechanical/electrical boundary conditions are transformed into the combination of the singularity exponents and characteristic angular functions, which are a set of characteristic ordinary differential equations with variable coefficients. The interpolating matrix method [28] developed by part of the authors before is applied to solving these variable coefficients equations in order to determine the characteristic values and characteristic vectors, which correspond precisely to the singularity exponents and characteristic angular functions. The present method is versatile for the V-notches in plane and out of plane loading cases. It can also be efficiently degenerated to calculate the singularity exponents and characteristic angular functions for the crack problems.

2. Characteristic equations for piezoelectric material containing V-notches under plane strain state

It is known that the piezoelectric–elastic problems can be decoupled into plane and out of plane ones according to the poling axis is parallel or perpendicular to the piezoelectric material surface. Let us consider a piezoelectric V-notch under plane strain state shown in Fig. 1a, where two Cartesian coordinate systems oxy, $ox'y'$ and a polar coordinate system $or\theta$ are defined. The polarization direction y' makes angle β with the y-axis, where β is measured from the y-axis in the counter-clockwise direction.

The constitutive equations for the plane field of a piezoelectric material in oxy coordinate system when the polarization direction is along the y-axis is given as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ D_x \\ D_y \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{13} & 0 & 0 & -e_{31} \\ C_{13} & C_{33} & 0 & 0 & -e_{33} \\ 0 & 0 & C_{44} & -e_{15} & 0 \\ 0 & 0 & e_{15} & k_{11} & 0 \\ e_{31} & e_{33} & 0 & 0 & k_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ E_x \\ E_y \end{Bmatrix} \tag{1}$$

where $(\sigma_x, \sigma_y, \tau_{xy})$ and $(\varepsilon_x, \varepsilon_y, \gamma_{xy})$ are respectively the stress and strain components, (D_x, D_y) and (E_x, E_y) are the components of the electric displacement and electric field strength, respectively. C_{11}, C_{13}, C_{33} and C_{44} are the elastic modulus, e_{13} and e_{15} are the piezoelectric constants, k_{11} and k_{33} are the dielectric constants. By the coordinate transformation, Eq. (1) can be rewritten in the polar coordinate system $(or\theta)$ as

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \\ D_r \\ D_\theta \end{Bmatrix} = \begin{bmatrix} Q_{11}(\theta-\beta) & Q_{12}(\theta-\beta) & Q_{13}(\theta-\beta) & Q_{14}(\theta-\beta) & Q_{15}(\theta-\beta) \\ Q_{21}(\theta-\beta) & Q_{22}(\theta-\beta) & Q_{23}(\theta-\beta) & Q_{24}(\theta-\beta) & Q_{25}(\theta-\beta) \\ Q_{31}(\theta-\beta) & Q_{32}(\theta-\beta) & Q_{33}(\theta-\beta) & Q_{34}(\theta-\beta) & Q_{35}(\theta-\beta) \\ Q_{41}(\theta-\beta) & Q_{42}(\theta-\beta) & Q_{43}(\theta-\beta) & Q_{44}(\theta-\beta) & Q_{45}(\theta-\beta) \\ Q_{51}(\theta-\beta) & Q_{52}(\theta-\beta) & Q_{53}(\theta-\beta) & Q_{54}(\theta-\beta) & Q_{55}(\theta-\beta) \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{r\theta} \\ E_r \\ E_\theta \end{Bmatrix} \tag{2}$$

where the matrix elements $Q_{ij}(\theta-\beta)$ $(i,j = 1, \cdots, 5)$ are the functions with respect to θ and β. The strain and electric field strength in Eq. (2) can be expressed by the displacement component u_r, u_θ and the electric potential ϕ like

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{r\theta} \\ E_r \\ E_\theta \end{Bmatrix} = \begin{Bmatrix} \partial u_r/\partial r \\ u_r/r + \partial u_\theta/(r\partial\theta) \\ \partial u_r/(r\partial\theta) + \partial u_\theta/(\partial r) - u_\theta/r \\ -\partial\phi/\partial r \\ -\partial\phi/(r\partial\theta) \end{Bmatrix} \tag{3}$$