Theoretical and Applied Fracture Mechanics 76 (2015) 75–90

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Comprehensive data for calculating the higher order terms of crack tip stress field in disk-type specimens under mixed mode loading

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ARTICLE INFO

Article history:
Available online 16 January 2015

Keywords:
Higher order terms
Finite element analysis
Fracture toughness
Mixed mode I/II loading
Semi-circular bend sample
Centrally cracked circular disk specimen

ABSTRACT

The aim of this paper is to determine the coefficients of singular, T-stress and third terms of crack tip stress field for two disk-type specimens under wide range combinations of mixed-mode I/II loading. The first specimen is a centrally cracked circular disk subjected to diametral load (CCCD sample) and the second one is a semi-circular disk under three-point bending (SCB sample). Using finite element method, the stress intensity factors K_I and K_{II} are calculated for two samples and then the state of pure mode II loading is expressed for each specimen. As an important parameter to predict the fracture toughness of quasi-brittle materials taking into account the geometry effect, the sign and magnitude of term A_3 are determined for both samples under pure mode I condition. It is found that the sign of A_3 is negative for any configurations of SCB sample under pure mode I while its sign for CCCD can be negative or positive corresponding to the ratio of a/R.

1. Introduction

Quasi-brittle materials such as rocks, concretes, ceramics, brittle polymers and glasses. usually contain numerous cracks. This is mainly because of their porous and granular microstructures. The crack, which plays the role of stress concentrator, processes the brittle fracture in components and structures made of quasi-brittle materials. Therefore, brittle fracture due to cracks should be considered in design and maintenance of structures and components. The linear elastic fracture mechanics (LEFM) investigates the brittle fracture of specimens in which the damaged zone around the crack tip is assumed to be relatively small and thus it is not considerable. On the other hand, the cracks are often subjected to different combinations of opening and sliding modes including pure mode I and pure mode II. Thus, the mixed mode I/II fracture behavior of quasi-brittle materials should be investigated in the practical applications. There are several criteria in LEFM such as the maximum tangential stress (MTS) criterion [1], the maximum energy release rate criterion [2] and the minimum strain energy density (SED) criterion [3]for predicting the onset of fracture in quasi-brittle materials under mixed mode loading. Moreover, there are many different test configurations to validate these criteria and also to investigate experimentally the mixed mode fracture of quasi-brittle materials. Single edge notched beam specimen under three or four-point bend [4–6], compact tension-shear specimen [7,8], the edge cracked triangular specimen under three-point bend loading [9] and the angled edge crack specimen subjected to a uniform far-field tension [10] are among the specimens used in the past for mixed-mode fracture tests. Since quasi-brittle materials are vulnerable to tensile loading, it is preferred to conduct fracture tests on specimens under compression. In this way, two disk-type specimens called centrally cracked circular disk (CCCD) [11–18] and the semi-circular bend (SCB) specimens [12,19–25] have been frequently used for mixed mode fracture testes of quasi-brittle materials. The CCCD specimen is a circular disk of radius R containing a centrally crack of length $2a$ under diametral compression as shown in Fig. 1a. The combination of mode I and II can be controlled in the CCCD specimens by changing the orientation of crack relative to the loading direction, i.e. by changing the crack angle α. Pure mode I loading in the CCCD specimen can be achieved easily by setting the direction of crack along the applied load ($\alpha = 0°$). When the crack inclination angle α reaches at specific angles corresponding to crack length ratios a/R, the crack flanks slide relative to each other without opening or compression and the CCCD specimen is loaded under pure mode II condition. The second sample which has been frequently used for fracture tests on quasi-brittle materials is a semi-circular disk of radius R containing an edge crack of length a subjected to three-point bending. In this specimen, the state of mode mixity can be controlled by one of these methods: (1) changing the orientation of crack relative to the applied load (see Fig. 1b), (2) shifting the vertical crack from the

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http://dx.doi.org/10.1016/j.tafmec.2015.01.004
0167-8442/

Nomenclature

a	crack length in SCB specimen and half-crack length in CCCD specimen
A_3, B_3	constant coefficients of the third terms in the Williams series expansion
A_3^*, B_3^*	non-dimensional forms of A_3 and B_3
CCCD	center cracked circular disk specimen
E	Young's modulus
FEA	finite element analysis
FEOD	finite element over-deterministic method
K_I	mode I stress intensity factor
K_{II}	mode II stress intensity factor
K_I^*, K_{II}^*	non-dimensional forms of K_I and K_{II}
LEFM	linear elastic fracture mechanics
MTS	maximum tangential stress criterion
MMTS	modified MTS criterion
P	applied load
R	radius of CCCD and SCB specimens
S	half the distance between the bottom supports in SCB samples
SED	strain energy density
r, θ	crack tip co-ordinate
t	specimen thickness
T	T-stress
T^*	non-dimensional form of T
u, v	displacement components in Cartesian coordinate system
α	crack inclination angle
κ	Kolosov constant
σ_{xx}, σ_{yy}, τ_{xy}	stress components in Cartesian coordinate system
ν	Poisson's ratio

applied load direction (see Fig. 1c) while the bottom supports are symmetry, (3) asymmetry distances of bottom supports from the vertical crack located at the middle of bottom edge (see Fig. 1d). For the sake of convenience, the SCB specimens subjected to these loading conditions are respectively denoted by SCB-A, SCB-B and SCB-C. Pure mode I loading takes place in all types of SCB sample when the vertical crack is along the direction of applied load and the distances of bottom supports are symmetry. Pure mode II loading will also occur at a specific inclination angle α^{II} for SCB-A sample while it takes place at the specific supporting distances from the vertical crack position for SCB-B and SCB-C.

Several studies in the past demonstrated that the classical fracture criteria (such as MTS, SED and G criteria) when are written only in terms of singular stress terms K_I and K_{II} cannot provide good estimates for the fracture test results [9,11,26,27]. It is also established that the modified form of fracture criteria which take into account the second and third terms of the Williams series expansion for describing the crack tip stresses can predict the experimental results more accurately [16,20,26,28–34]. For instance, Ayatollahi and Sedighiani [16] showed that the modified SED criterion which makes use of the T-stress term can provide good estimates for mode I fracture behavior of Guiting limestone obtained from the SCB and CCCD specimens. As another example, Ayatollahi and Akbardoost [26] used the modified MTS criterion which considers the effects of the second and third stress terms in addition to the singular terms for predicting the fracture behavior of rocks measured from the SCB and CCCD samples. Chao et al. [32], Karihaloo [35] and Dyskin [33] are among the researchers who underlined the effects of non-singular stress terms in fracture behavior analysis of quasi-brittle materials. Therefore, the higher

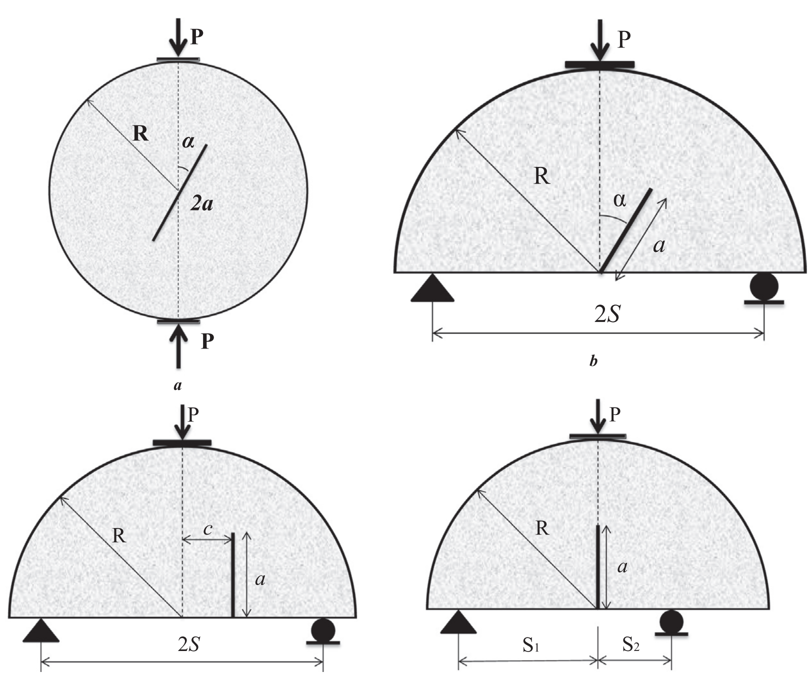

Fig. 1. The schematics of disc-type specimens and their mixed mode loading configurations, (a) CCCD, (b) SCB-A, (c) SCB-B, (d) SCB-C.