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The interaction between an edge dislocation and a semi-infinite long crack penetrating a circular inhomogeneity

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ABSTRACT

The interaction between an edge dislocation and a semi-infinite long crack penetrating a circular inhomogeneity is investigated. Using Riemann–Schwartz's symmetry principle integrated with the analysis of singularity of complex function, the closed form solutions of complex potentials and stress field are obtained. Stress intensity factor at the crack tip and image force acting on edge dislocation are calculated. As a result, the crack penetrating inhomogeneity and interface has a strong influence on the interaction between an edge dislocation and an elastic circular inhomogeneity. It attracts the edge dislocation evidently and because of the influence of crack, the edge dislocation has a stable equilibrium position in matrix. Under a certain condition, the edge dislocation can reduce the stress intensity factor near crack tip. If the distance between the edge dislocation and crack tip is fixed, there always exists a critical value of position angle of edge dislocation at which the shielding or anti-shielding effect is maximal. Shielding effect to the stress intensity factor increases when edge dislocation approaches the tip of crack. In addition, the material mismatch also has a great influence on image force, shielding and anti-shielding effects.

1. Introduction

Reinforced fibers or particles in composite materials can be regarded as inhomogeneities or inclusions with respect to the matrix material [1]. Edge cracks may penetrate inhomogeneities and interfaces and form penetrable continuous cracks. Therefore, the study of interaction among dislocation, inhomogeneity and crack penetrating the interface has important theoretical significance and practical value to understand the fracture mechanism and the plastic phase toughening mechanism in composites and the shielding effect of the crack tip. In recent years, intensive researches have been conducted in this area [2–36].

More specifically, Fang and Liu [25] studied the interaction between an edge dislocation and a nanoscale inhomogeneity with interface effects, which shows the edge dislocation can be attracted (repelled) by a hard (soft) circular inhomogeneity with interface stress. The problem concerning elastic fields due to an edge dislocation in an isotropic film-substrate by the image method was investigated by Zhou and Wu [26]. Zhang et al. [27] studied the interaction between edge dislocation and semi-infinite wedge crack and obtained the closed-form expressions of the stress intensity factor at crack tip, image force and strain energy of dislocation, which shows that the radial force of an edge dislocation is inversely proportional to the distance from the crack tip to the dislocation and is independent of the wedge angle. Zhang and Li [28] analyzed the interaction between edge dislocation and semi-infinite interfacial crack penetrating interface, which indicates that the image force exerted on an edge dislocation is inversely proportional to the distance from the crack tip to the dislocation and the proportional constant is controlled by the shear modulus of the softer phase. The effects of elastic mismatch and crack tip position on the stress intensity factor of a long crack penetrating a circular inhomogeneity were solved by Wang and Roberto [29]. Zhou and Wei [30] studied the interactions among multiple cracks and inclusions near surfaces under contact loading using Eshelby's equivalent inclusion method and the eigenstrain approach. Steif [31] obtained the closed-form expression of the stress intensity factor at the tip of a semi-infinite crack partially penetrating a circular inclusion by dislocation integral method. Song and Gao [32] investigated problem concerning the screw dislocation near a semi-infinite wedge crack tip inside nano-circular inclusion. The stress intensity factors, image force, as well as the critical loads for dislocation emission were discussed in detail. Wang et al.

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[33] studied the interaction between screw dislocation and a semi-infinite crack penetrating a piezoelectric circular inhomogeneity with a viscous interface and obtained the closed-form solutions under three loading cases. The emission criterion of a screw dislocation from a semi-infinite wedge crack penetrating an inhomogeneity was discussed by Zhang et al. [34]. In addition, emission criterion of a screw dislocation near a semi-infinite wedge crack partially penetrating a nanoscale piezoelectric inhomogeneity with interface effect was also investigated by Zhang et al. [35]. Recently, the problem of the interaction among edge dislocation, Griffith crack and circular inclusion was studied by Tao et al. [36]. However, the problem of edge dislocation interacting with a semi-infinite long crack penetrating a circular inhomogeneity has not been studied due to the complexity of the calculation.

In the present paper, the problem of the elastic interaction between an edge dislocation and a semi-infinite long crack penetrating a circular inhomogeneity is investigated by the conformal mapping technique and the complex variable method. The image force acting on edge dislocation and the stress intensity factor at the tip of the crack are derived. The impacts of material mismatch and the position of dislocation on the image force and shielding effect to the crack are discussed.

2. Problem statement and solution

As shown in Fig. 1, an infinite elastic matrix contains a circular inhomogeneity of radius R_0. The domains occupied by the matrix and the inhomogeneity are denoted by S^- and S^+, respectively. Both matrix and inhomogeneity are assumed to be isotropic elastic. The shear modulus and poisson's ratio of the matrix and inhomogeneity are given by μ_i and v_i ($i = 1, 2$), respectively, where '1' denotes the inhomogeneity and '2' denotes the matrix. Perfect bonding condition is assumed at the matrix-inhomogeneity interface. An edge dislocation ($b_x + ib_y$) is located at arbitrary point $z_0 = x_0 + iy_0$ ($z_0 = re^{i\theta}$) in matrix. A semi-infinite straight line crack with 0° crack opening angle partially penetrates through the interface L_0 and the upper and lower surfaces of the crack are traction-free surfaces. Here, the line crack starts at the center of the inhomogeneity, and Cartesian and polar coordinate systems are established with their origins at the crack tip. The intersection region of the inhomogeneity, the matrix and the crack surface is a bi-material wedge with 90° angle, whose tip exhibits stress singularity. Previous studies [37,38] have presented many approaches to the solution of these singular points, based on the concept of proper boundary-value problem and the theorem of homogeneous solutions. In order to simplify the analysis, the stress singularity of the meeting points of the inhomogeneity, the matrix and the crack surfaces (the tip of the bi-material wedge) is not considered temporarily in the present study.

The mapping function is given as follows

$$z = \omega(\zeta) = \zeta^2 \tag{1}$$

Using the mapping function, the edge crack in the z-plane maps into a half space in the ζ-plane ($\xi > 0$), where ξ and η are coordinates with the origin located at the crack tip, as shown in Fig. 1(b). In addition, the cracked circular inhomogeneity is mapped onto the half-circular region $|\zeta| < R$ and $Re[\zeta] \geqslant 0$ in the ζ-plane.

For a plane problem, the stress and the corresponding boundary conditions can be expressed in terms of the two well-known Muskhelishvili's complex potentials $\phi(z)$ and $\psi(z)$ [39]

$$\sigma_{xx} + \sigma_{yy} = 2[\phi'(z) + \overline{\phi'(z)}] \tag{2}$$

$$\sigma_{yy} - i\sigma_{xy} = \phi'(z) + \overline{\phi'(z)} + z\overline{\phi''(z)} + \overline{\psi'(z)} \tag{3}$$

$$p_x + p_y = (-i)[\phi(z) + z\overline{\phi'(z)} + \overline{\psi(z)}]_A^B \tag{4}$$

$$2\mu(u_x + iu_y) = \kappa\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)} \tag{5}$$

where $z = x + iy$ is the complex coordinate, p_x and p_y are the resultant force components along the x and y directions acting on the arbitrary segment AB of crack, $\kappa = 3{-}4v$ for plane strain.

Using the conformal mapping approach, Eqs. (2)–(5) can be rewritten in the ζ-plane as follows

$$\sigma_{xx} + \sigma_{yy} = 2\left[\frac{\phi'(\zeta)}{\omega'(\zeta)} + \overline{\frac{\phi'(\zeta)}{\omega'(\zeta)}}\right] \tag{6}$$

$$\sigma_{yy} - i\sigma_{xy} = \frac{\phi'(\zeta)}{\omega'(\zeta)} + \frac{\overline{\phi'(\zeta)}}{\overline{\omega'(\zeta)}} + \omega'(\zeta)\overline{\left(\frac{\phi'(\zeta)}{\omega'(\zeta)}\right)'} + \frac{\overline{\psi'(\zeta)}}{\overline{\omega'(\zeta)}} \tag{7}$$

$$p_x + p_y = (-i)\left[\phi(\zeta) + \omega(\zeta)\frac{\overline{\phi'(\zeta)}}{\overline{\omega'(\zeta)}} + \overline{\psi(\zeta)}\right]_A^B \tag{8}$$

$$2\mu(u_x + iu_y) = \kappa\phi(\zeta) - \omega(\zeta)\frac{\overline{\phi'(\zeta)}}{\overline{\omega'(\zeta)}} - \overline{\psi(\zeta)} \tag{9}$$

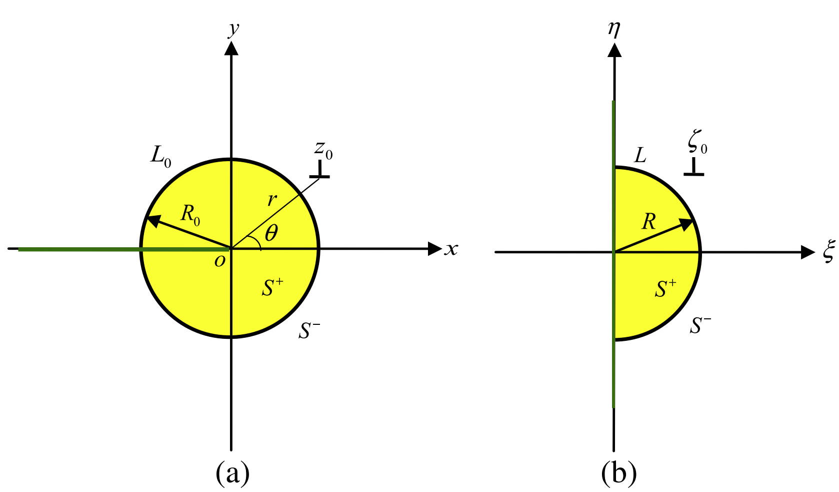

Fig. 1. An edge dislocation embedded in infinite matrix with a long crack penetrating a circular inhomogeneity. (a) Physical plane and (b) conformal mapping.