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A meshfree variational multiscale methods for thermo-mechanical material failure

H.S. Wang

S China Univ Technol, State Key Lab Subtrop Bldg Sci, Guangzhou 510640, People's Republic of China

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ABSTRACT

A meshless variational multiscale methods for thermo-mechanical material failure is presented. Fracture is modeled by partition of unity enrichment. The displacement field and temperature field is enriched with step-functions and appropriate crack tip enrichment accounting for fine-scale features. The topology of the crack is modeled taking advantage of the level set method. The advantage of using a meshless method instead of finite elements is the ease in treating highly curved cracks with very coarse meshes due to the higher continuity of the meshless method. Moreover, the higher continuity results also in a smoother and more accurate stress field avoiding eratic fracture patterns. The method is applied to several benchmark problems and compared to analytical results, reference solutions and experimental data to demonstrate its robustness and efficiency.

1. Introduction

Thermo-mechanical problems for fracturing solids are of high relevance in many applications of Engineering Science. While there are various computational methods for mechanical fracture problems [5,15,13,30,35,81,37,97,67,63,34,64,21,50,52,83,87,91,92,93,94,95,96], there are comparatively few contributions dealing with the simulation of material failure of such coupled problems. Classically, the finite element method has been used to model this type of problems. However, finite element method is not well suited for discrete material failure as it requires aligning the crack to the element edges that makes the computational results sensitive with respect to the discretization. Meshless methods [51] are an interesting alternative to finite element methods [36,9,7,55,38,2,3,6,4], smoothed finite elements [19,20,89,24,57], extended finite element methods [48,11,10] or related methods [85,31,26,48] such as the phantom node method [84,90,73,24] due to the following advantages:

- Meshless methods are higher order continuous. It has been shown numerically that this property is advantageous [76,70,18,72] as the stress field is less eratic in front of the crack tip and curved cracks can be modeled with coarser discretizations [43,45,44,41,42,79,80,71,58]. Though XFEM can model crack growth without remeshing, it still requires a certain mesh refinement in front of the crack tip.
- H-adaptivity can be incorporates much easier in a meshless method due to the absence of a mesh [60,65,47].
- Meshless methods are more suitable for dynamic fracture and fragmentation and problems involving finite strains and large deformations. They do not require any element deletion that causes errors in the energy balance and results in loss of mass.

Among many meshless methods, the element-free Galerkin method (EFGM) [14,12] has been one of the most established and widely used one. Discrete fracture has been modeled in the EFGM by concepts such as the visibility method and subsequent improvements such as the transparency method or diffraction method [54,17]. Partition-of-unity enrichment has been used in the context of meshless methods before the development of XFEM [30]. A very elegant approach to fracture was proposed by [27,29] who enriched the weight functions instead of the entire approximation; see also the recent work by [8]. It facilitates the implementation as no additional degrees of freedom (DOFs) need to be introduced. However, introducing the *exact* information of the near-crack tip displacement field into this approach seems to be more difficult. The extension of XFEM into a meshless concept was done by [88]. The approach was extended by [1] to LME shape functions and [66] to non-linear problems and thin shell analysis [69] as well as fluid–structure interaction [77]. [18,76,97] suggested to omit crack tip enrichment functions for applications in two- and three-dimensional non-linear fracture applications as the implementation of the crack tip (or crack front in 3D) enrichment complicates the implementation strategy. Moreover, for non-linear

E-mail address: sulin.wang@aol.com

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problems the near-crack-tip solution is not known. However, one difficulty is to enforce appropriate crack closure which was realized by the Lagrange multiplier method adding additional complexity. Another efficient path to fracture is the Cracking Particle Method (CPM) [59]. The CPM models the crack topology as a set of plane crack segments crossing the entire domain of influence of the cracked node. The CPM has been extended to ductile fracture [71,65], large deformations [62] and was applied to interesting problems in statics [61,86,25] and dynamics [74,23,33,75,78].

Here, we present for the first time the extended element-free Galerkin method (XEFGM) for thermo-mechanical fracture in linear elastic solids. It can be regarded as an extension of the work of [28] for adiabatic cracks to meshless methods. The structure of this paper is as follows: After a brief introduction to EFG, the governing equations and the weak form will be discussed first. Then, the discretization of the displacement and temperature field will be explained. Subsequently, we will derive the discrete system of equations and present several benchmark examples before the manuscript concludes with a summary and future research perspectives.

2. Element-Free Galerkin Method (EFGM)

We briefly summarize the EFGM-approximation [14] given by

$$u_i(\mathbf{x}) = \sum_{I\in\mathcal{W}} p_I(\mathbf{x})\ a_I(\mathbf{x}) = \mathbf{p}\ \mathbf{a} \tag{1}$$

$p_I(\mathbf{x})[1\ x\ y]$ denoting the linear polynomial basis and $a_I(\mathbf{x})$ denoting the unknowns; the set of nodes in the discretization is denoted by $\mathcal{W}$. Minimization of the discrete error norm $\mathcal{J}$ with respect to (w.r.t.) to the unknown coefficients $\mathbf{a}$

$$\mathcal{J}(\mathbf{a}) = \sum_{I\in\mathcal{W}} \left(\mathbf{p}^T(\mathbf{x}_I)\ \mathbf{a}(\mathbf{x}_I) - \mathbf{u}_I\right)^2 w(\mathbf{x}-\mathbf{x}_I, h) \tag{2}$$

yields the EFG-approximation

$$u_i(\mathbf{x}) = \sum_{I\in\mathcal{W}} N_I(\mathbf{x}) u_{iI}(t) \tag{3}$$

It can be shown that the shape functions are given by

$$N_I(\mathbf{x}) = \mathbf{p}^T(\mathbf{x})\ A^{-1}(\mathbf{x})\ \mathbf{D}_I(\mathbf{x}) \tag{4}$$

with

$$\begin{aligned}\mathbf{D}_I(\mathbf{x}) &= w(\mathbf{x}-\mathbf{x}_I, h)\mathbf{p}^T(\mathbf{x}_I)\\ \mathbf{A}_I(\mathbf{x}) &= \sum_{I\in\mathcal{W}} w(\mathbf{x}-\mathbf{x}_I, h)\ \mathbf{p}(\mathbf{x}_I)\ \mathbf{p}^T(\mathbf{x}_I)\end{aligned} \tag{5}$$

where $w(\mathbf{x}-\mathbf{x}_I, h)$ is the kernel function and h a parameter governing the support size.

3. Governing equations and weak form

Let us consider domain Ω with boundary $\Gamma = \Gamma_u \bigcup \Gamma_t \bigcup \Gamma_c = \Gamma_T \bigcup \Gamma_q$ and $\Gamma_u \bigcap \Gamma_t = 0, \Gamma_c \bigcap \Gamma_t = 0, \Gamma_u \bigcap \Gamma_c = 0; \Gamma_T$ denotes the boundary where temperature is imposed, flux is imposed on Γ_q, Γ_t and Γ_u are von Neumann boundary and Dirichlet boundary, respectively, and Γ_c denotes the crack boundary. In this study, we consider only adiabatic cracks, i.e. $\Gamma_c \subset \Gamma_q;\ \bar{q} = 0\ on\ \Gamma_c$.

The equilibrium equation and the heat equation in strong form is expressed by

$$\begin{aligned}\sigma_{ij,j} + b_i &= 0 \quad \forall \mathbf{x} \in \Omega\\ q_{i,i} + Q &= 0 \quad \forall \mathbf{x} \in \Omega\end{aligned} \tag{6}$$

σ_{ij} denoting stress tensor, $\mathbf{b}_i$ the body forces, Q is heat source and q_i denotes the heat flux. For problems in thermo-elasticity, the compatibility conditions and the constitutive equations are written as

$$\begin{aligned}q_i + kT_{,i} &= 0_i\\ \epsilon_{ij} &= \underbrace{\frac{1}{2}\left(u_{i,j} + u_{j,i}\right)}_{\epsilon_{ij}^{mech}} - \underbrace{\alpha_T \Delta T I_{ij}}_{\epsilon_{ij}^{therm}}\\ \sigma_{ij} &= C_{ijkl}\epsilon_{kl}\end{aligned} \tag{7}$$

where the upper indices *therm* and *mech* designate thermal and mechanical quantities, respectively; ΔT is the change in temperature with $\Delta T = T - T_0, T_0$ denoting room temperature and I_{ij} is the second order identity matrix. The first order elasticity tensor is denoted by C_{ijkl} and ϵ_{ij} is the (small) strain tensor; α_T is the expansion coefficient, k the diffusivity. Note that the thermal field influences the mechanical field while there is no influence of the mechanical field on the thermal field. In other words, the thermal field could be solved decoupled from the mechanical field. However, in this manuscript we opt to solve the system of equations in a coupled way. The boundary conditions are given by

$$\begin{aligned}u_i &= \bar{u}_i\ on\ \Gamma_u\\ T &= \overline{T}\ on\ \Gamma_T\\ t_i &= \sigma_{ij}n_j = \bar{t}_i\ on\ \Gamma_t\\ q_i n_i &= \bar{q}\ on\ \Gamma_q\end{aligned} \tag{8}$$

We assume that the tractions are zero at the free boundary and at the crack surface. The superimposed bar indicates imposed values at the associated boundary. The weak form is obtained by multiplying the governing Eq. (6) with arbitrary admissible test functions $\delta u_i \in \mathcal{U}_0$ and $\delta T \in \mathcal{T}_0$ and integrating over volume Ω. Performing integration by parts and with divergence theorem, we obtain the weak form of governing equations:

$$\begin{aligned}&\int_\Omega \delta\epsilon_{ij}^{mech}\sigma_{ij}\ d\Omega - \int_\Omega \delta u_i b_i\ d\Omega - \int_{\Gamma_t} \delta u_i \bar{t}_i\ d\Gamma = 0\\ &\int_\Omega \delta q_i k q_i\ d\Omega + \int_\Omega \delta T Q\ d\Omega - \int_{\Gamma_q} \delta T \bar{q}\ d\Gamma = 0\end{aligned} \tag{9}$$

The solution spaces of admissible test functions δu_i and δT and trial functions $u_i \in \mathcal{U}$ and $T \in \mathcal{T}$ are given by

$$\begin{aligned}\mathcal{U} &= \left\{u_i | u_i \in \mathcal{H}^1,\ \ u_i = \bar{u}_i\ \ on\ \ \Gamma_u,\ u_i\ discontinuous\ on\ \Gamma_c\right\}\\ \mathcal{U}_0 &= \left\{\delta u_i | \delta u_i \in \mathcal{H}^1,\ \ \delta u_i = 0_i\ \ on\ \ \Gamma_u,\ \delta u_i\ discontinuous\ on\ \Gamma_c\right\}\\ \mathcal{T} &= \left\{T | T \in \mathcal{H}^1,\ \ T = \overline{T}_i\ \ on\ \ \Gamma_T,\ T\ discontinuous\ on\ \Gamma_T\right\}\\ \mathcal{T}_0 &= \left\{\delta T | \delta T \in \mathcal{H}^1,\ \ \delta T = 0\ \ on\ \ \Gamma_T,\ \delta T\ discontinuous\ on\ \Gamma_T\right\}\end{aligned} \tag{10}$$

$\mathcal{H}$ denoting the first Sobolev space.

As mentioned previously, the non-interpolatory character the EFG approximation complicates the imposition of Dirichlet boundary conditions. Classical approaches to impose Dirichlet boundary conditions include coupling to finite element method [16,39,68] where finite elements are used close to Dirichlet boundaries and therefore boundary conditions can then be imposed on the FE-boundary and the Lagrange multiplier method [17]. In this manuscript, we use the boundary collocation technique as described in [32] due to its simplicity and robustness. Details can be found in the given reference. An excellent overview on meshless methods including imposition of boundary conditions is the paper by [51].

4. Discretization

4.1. Displacement field

The displacement field can be decomposed into a continuous or standard part $\mathbf{u}^{st}$ and discontinuous or enriched part $\mathbf{u}^{en}$:

$$\begin{aligned}u_i^h(\mathbf{x}) &= u_i^{st}(\mathbf{x}) + u_i^{en}(\mathbf{x})\\ \delta u_i^h(\mathbf{x}) &= \delta u_i^{st}(\mathbf{x}) + \delta u_i^{en}(\mathbf{x})\end{aligned} \tag{11}$$