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Modeling of dynamic fracture based on the cracking particles method

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ABSTRACT

The dynamic fragmentation of brittle materials using the cracking particles method (CPM) with obscuration zone is studied. The CPM is an effective meshfree method for arbitrary evolving cracks. The crack is modeled by piecewise straight crack segments and does not require any topological representation of the crack surface. To avoid artificial cracks observed in discrete continuum approaches, obscuration zones are used as suggested by Mott. The influence of the variation in material properties with different stochastic input parameters on the results is studied as well.

1. Introduction

Modeling fragmentation remains one of the key challenges for analytical and computational methods. Statistical models for fragmentation were developed already in the 1940s by [1,2]. He assumed a statistical distribution of the fracture strain and considered obscuration zones evolving around propagating and initiating cracks. One of his key findings was the dependence of the strain-rate, the homogeneity of the material and the propagation speed of the obscuration zones on the fragment sizes. The more heterogeneous the material, the larger the fragment sizes. [3] considered later different statistical distributions and found that theories purely based on statistics cannot appropriately model the complicated physical fragmentation phenomenon.

Refs. [4,5] was one of the first researchers combining mechanical and stochastic models. In the context of a damage model based on the micro-structure of the material, they assumed statistical distributions for initiating and propagating micro-cracks. They implemented their concept into finite element software and obtained fragment distributions that closely match experimental observations. Similar studies were conducted by [6,7] but they did not consider fine-scale features of the micro-structure. They predicted statistical distribution of fragment sizes in dependence of macroscopic material parameters such as damage. However, studies by [8] revealed that fragments contain internal damage and therefore the damage alone should not be used as criterion for fragment sizes. Other studies [6,9–13] take advantage of energy concepts to predict fragment sizes. Most of the fragmentation models rely on numerical methods to predict mechanical properties or quantities. Commonly, finite element analysis was performed not considering the fragmentation process explicitly. However, advances in computational methods provide now promising alternatives to model the fragmentation process directly. One promising alternative to the FEM are meshless methods [14–16].

SPH (Smoothed Particle Hydrodynamics) simulations [17] predicted quite accurately fragment distributions and fragment sizes. Due to the meshfree character of the computational method [15,16,18,19], these simulations were able to model the fragmentation process explicitly. Other simulations on dynamic fracture and fragmentation for brittle or quasi-brittle materials were performed by [20–23]. Artificial fracture has been one concern in modeling fracture and fragmentation in early meshfree methods but those issues have been resolved now [24–26]. Meshfree methods for discrete crack approaches have been proposed for example by [27–29], alternative approaches are for instances based on phase fields [30,31], remeshing based on edge rotation [32–38], or multiscale approaches [39–43] or others [44–47]. A very promising approach for fragmentation is the cracking particles method (CPM) developed by [48]. The CPM does not require any representation of the crack surface. The crack is modeled by a set of crack segments that pass through the nodes. Hence, fragmentation is a natural outcome of the simulation. The CPM was developed in three-dimensions and extended to fracture in shells [49,50] including FSI (fluid–structure-interaction) [51–53], ductile fracture [54–56] and other problems [57–71].

We propose a model for fragmentation combining the CPM and obscuration zones. We will determine the influence of the strain date, initial crack distribution on the fragment size. We also check the validity of the hypothesis of obscuration zones by numerical simulations. The manuscript is structured into 6 parts: the cracking

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particles method, the damage model and cohesive zone model, the model for fragmentation based on obscuration zoned, numerical examples and concluding remarks.

2. Cracking particle method

The CPM [48] is a partition-of-unity meshfree method where the displacement field is decomposed in a standard part $\mathbf{u}^{st}$ and in an enriched part $\mathbf{u}^{en}$

$$\mathbf{u}(\mathbf{X},t) = \mathbf{u}^{st}(\mathbf{X},t) + \mathbf{u}^{en}(\mathbf{X},t) \tag{1}$$

where t is the time and $\mathbf{X}$ are material coordinates. The original CPM is based on Petrov–Galerkin corrected derivatives method [24] but it was found advantageous to use MLS-shape functions in the context of the element-free Galerkin (EFG)-method [18] as it was also proposed by [50,72]. The MLS approximation is used for both, the standard part and the enriched part of the displacement field. The MLS approximant is defined by

$$\mathbf{u}^{st}(\mathbf{X},t) = \sum_{I\in\mathcal{W}} p_I(\mathbf{X})\ a_I(\mathbf{X},t) \tag{2}$$

where $a_I(\mathbf{X},t)$ denotes the unknown coefficients, $p_I(\mathbf{X})$ is the basis vector and $\mathcal{W}$ is the set of nodes in the entire discretization. The MLS approach can also be written in matrix form:

$$\mathbf{u}^{st}(\mathbf{X},t) = \mathbf{p}(\mathbf{X})\mathbf{a}(\mathbf{X}) \tag{3}$$

In order to guarantee first-order completeness, linear functions $\mathbf{p}(\mathbf{X}) = [1 X Y]^T$ for two-dimensional problems are used. As it was shown for example by [24], it is important for stability reasons to formulate the MLS approximation in terms of material coordinates $\mathbf{X}$ instead of spatial coordinates $\mathbf{x}$ commonly done in SPH-approaches [17,20,73,74]. The MLS approximation is obtained by minimizing a discrete weighted $\mathcal{L}_2$ norm $\mathcal{L}$ with respect to the unknown coefficients $\mathbf{a}$

$$\mathcal{L} = \sum_{I\in\mathcal{W}} \left(\mathbf{p}^T(\mathbf{X}_I)\ \mathbf{a}(\mathbf{X}_I,t) - \mathbf{u}_I(t)\right)^2 w(\mathbf{X}-\mathbf{X}_I,h_0) \tag{4}$$

that finally leads to the famous MLS approximation

$$\mathbf{u}^{st}(\mathbf{X}) = \sum_{I\in\mathcal{W}} N_I(\mathbf{X})\mathbf{u}_I(t) \tag{5}$$

with

$$N_I(\mathbf{X}) = \mathbf{p}^T(\mathbf{X})\ A^{-1}(\mathbf{X})\ \mathbf{D}_I(\mathbf{X}) \tag{6}$$

and

$$\begin{aligned}\mathbf{D}_I(\mathbf{X}) &= w(\mathbf{X}-\mathbf{X}_I,h_0)\mathbf{p}^T(\mathbf{X}_I)\\ \mathbf{A}_I(\mathbf{X}) &= \sum_{I\in\mathcal{W}} w(\mathbf{X}-\mathbf{X}_I,h_0)\ \mathbf{p}(\mathbf{X}_I)\ \mathbf{p}^T(\mathbf{X}_I)\end{aligned} \tag{7}$$

where $w(\mathbf{x}-\mathbf{x}_I,h_0)$ denotes the kernel function and h_0 is its dilation parameter with respect to the reference configuration. In the presented studies, the cubic B-spline commonly used in meshfree methods is employed

$$w(s) = \begin{cases} \frac{2}{3} - 4s^2 + 4s^3 & 0 \leqslant s \leqslant \frac{1}{2} \\ \frac{4}{3} - 4s + 4s^2 - \frac{4}{3}s^3 & \frac{1}{2} \leqslant s \leqslant 1 \\ 0 & s > 1 \end{cases} \tag{8}$$

expressed in terms of the normalized and shifted parameter $s = \frac{\mathbf{X}-\mathbf{X}_I}{h_0}$.

The basic idea of the enriched part of the displacement field is to extend the 'standard' approximation in order to capture the kinematics introduced by cracks. Therefore, the standard shape functions $N_I(\mathbf{X})$ are replaced by enriched shape functions $N_I^{en}(\mathbf{X})$. The enriched shape functions consists of two parts, a product between the standard shape functions $N_I(\mathbf{X})$ and an enrichment function $\psi(\mathbf{X})$ accounting for the kinematics of the crack. Therefore, the enriched approximation of the displacement field reads:

$$\mathbf{u}^{en} = \sum_{I\mathcal{W}^{en}} N_I^{en}(\mathbf{X})\tilde{\mathbf{u}}_I = N_I(\mathbf{X})\Psi(\mathbf{X})\tilde{\mathbf{u}}_I \tag{9}$$

with additional degrees of freedom $\tilde{\mathbf{u}}_I$ and enrichment function

$$\psi(\mathbf{X}) = \begin{cases} 1 & (\mathbf{X}-\mathbf{X}_C)\cdot\mathbf{n} > 0 \\ -1 & (\mathbf{X}-\mathbf{X}_C)\cdot\mathbf{n} < 0 \end{cases} \tag{10}$$

accounting for the discontinuous displacement field; $\mathbf{X}_C$ is a point on the crack surface and $\mathbf{n}$ denotes the normal to the crack surface. Only the cracked particles in the set $\mathcal{W}^{en}$ are enriched. The crack crosses the entire domain of influence of the associated particle.

The first manuscript of the CPM suggested the use of a Petrov–Galerkin method based on stress-point point integration. Meanwhile, several improvements have appeared concerning the kinematic description of the crack surface up to cracking rules in order to avoid spurious cracking [59–63,75]. We have adopted the approach in [75]. In this paper, the CPM is used in the context of the EFG-method and a Bubnov–Galerkin method.

3. Constitutive models

3.1. Continuum model and fracture criterion

The material in the bulk is modeled with a scalar damage model:

$$\sigma = (1-D)\mathbf{C} : \epsilon \tag{11}$$

with the fourth-order elasticity tensor $\mathbf{C}$ and the damage variable D that depends on an effective strain $\tilde{\epsilon}$:

$$D(\tilde{\epsilon}) = 1(1-A)\ \epsilon_0\tilde{\epsilon}^{-1} - Ae^{-B(\tilde{\epsilon}-\epsilon_0)} \tag{12}$$

with material parameters A, B and ϵ_0 and

$$\tilde{\epsilon} = \sqrt{\sum_{j=1}^{3} H(\epsilon_j)\ \epsilon_j^2} \tag{13}$$

$H(\epsilon_j)$ being the Heaviside function and ϵ_j are the principal strains.

The transition from the continuum to discontinuum is modeled by loss-of-material-stability criterion [48,72]. Therefore, we compute the eigenvalues of the acoustic tensor $\mathbf{Q} = \mathbf{N}\cdot\mathbf{C}\cdot\mathbf{N}; \mathbf{N}$ being the normal to the crack surface when the eigenvalue is smaller or equal to zero and $\mathbf{C}$ is the tangent stiffness matrix of the material. The loss-of-material-stability analysis provides directly the orientation of the crack surface in contrast to other criteria such as the Rankine criterion where the crack is often introduced parallel to the direction of the maximum principal tensile stress.

As it is well known that local stress tensor does not provide adequate results, a non-local stress tensor is used [76–78]:

$$\sigma^{nl} = \frac{\sum_I \sigma w_I(\mathbf{X})}{\sum_I w_I(\mathbf{X})} \tag{14}$$

where the term in the denominator accounts for boundary effects. We use the same averaging (=kernel) function $w_I(\mathbf{X})$ as for the MLS approximation.

3.2. Cohesive zone model

The cohesive zone model (CZM) of [79] is used taking into account tangential as well as normal cohesive tractions. Therefore, let us introduce a so-called effective crack opening displacement

$$[[u]]_{eff} = \sqrt{[[u]]_t^2 + <[[u]]_n>^2} \tag{15}$$