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Two collinear square-hole cracks in an infinite plate in tension

Changqing Miao [a], Yintao Wei [b], Xiangqiao Yan [a,*]

[a] Res Lab on Composite Materials, Harbin Inst Technol, Harbin 150001, PR China
[b] Dept Automot Engn, Tsinghua University, Beijing 100084, PR China

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ABSTRACT

By using a hybrid displacement discontinuity method and a generalization of Bueckner's principle, the interactions of two collinear square-hole cracks in an infinite plate in tension are investigated in this paper. Numerical examples are included to illustrate that the numerical approach is very effective for analyzing multiple-hole cracks in an infinite plate in tension. Many numerical results are given and discussed. It is found that a square hole has a shielding effect on crack(s) emanating from the hole. The finding perhaps has an important meaning in engineering.

1. Introduction

Due to the stress concentration effect around the hole, cracks are likely to initiate at the hole under the action of fatigue loading. Consequently, a number of papers dealing with hole edge crack problems are available, see Ref. [1]. For radial crack(s) emanating from a circular hole in an infinite plate under tension, typical solutions were given by Bowie [2] and Newman [3]. For radial cracks emanating from an elliptical hole in an infinite plate under tension, typical solutions were obtained by Nisitani and Isida [4], Murakami [5] by using the body force and by Newman [3] by the boundary collocation method. For cracks emanating from a triangular or square hole in an infinite plate under tension, Murakami [5] used the body force method to calculate their stress intensity factors.

By using a hybrid displacement discontinuity method [7] and a generalization of Bueckner's principle [6], this paper concerns with interactions of two collinear square-hole cracks in an infinite plate in tension, shown in Fig. 1. Many numerical results are given and discussed. It is found that a square hole has a shielding of crack(s) emanating from the hole. These finding perhaps has an important meaning in engineering.

2. A generalization of Bueckner's principle

In 1958, Bueckner [6] derived an important result, which is related to the principle of superposition. He demonstrated the equivalence of the SIFs resulting from external loading on a body and those resulting from internal tractions on the crack face. The SIFs for a crack in a loaded body may be determined by considering the crack to be in an unloaded body with applied tractions on the crack surface only. These surface tractions are equal in magnitude but opposite in sign to those evaluated along the line of the crack site in the uncracked configuration.

Recently, Yan [9] tried to extend Bueckner's principle suited for a crack to a hole crack problem in infinite plate subjected to subjected to remote loads, σ_{xx}^{∞}, σ_{yy}^{∞} and σ_{xy}^{∞}, called an original problem for short. The original problem is divided into a homogeneous problem (the one without hole and cracks) subjected to remote loads and a hole-crack problem in infinite plate in an unloaded body with applied tractions on the crack surface and the hole surface. The applied tractions on the crack surface are equal in magnitude but opposite in sign to those evaluated along the line of the crack site in the uncracked configuration, which are

$$\begin{aligned}\sigma_{nn} &= \sigma_{xx}^{\infty}\sin\phi\sin\phi - 2\sigma_{xy}^{\infty}\sin\phi\cos\phi + \sigma_{yy}^{\infty}\cos\phi\cos\phi),\\ \sigma_{ns} &= \left(\sigma_{yy}^{\infty} - \sigma_{xx}^{\infty}\right)\sin\phi\cos\phi + \sigma_{xy}^{\infty}(\cos\phi\cos\phi - \sin\phi\sin\phi),\end{aligned} \tag{1a}$$

in which ϕ is the orientation angle of the line of the crack with respect to the x axis. Denoting the orientation angle of a tangent at any point on the surface of the hole with respect to the x axis by γ, the applied tractions at this point are

$$\begin{aligned}\sigma_{nn} &= \sigma_{xx}^{\infty}\sin\gamma\sin\gamma - 2\sigma_{xy}^{\infty}\sin\gamma\cos\gamma + \sigma_{yy}^{\infty}\cos\gamma\cos\gamma),\\ \sigma_{ns} &= \left(\sigma_{yy}^{\infty} - \sigma_{xx}^{\infty}\right)\sin\gamma\cos\gamma + \sigma_{xy}^{\infty}(\cos\gamma\cos\gamma - \sin\gamma\sin\gamma).\end{aligned} \tag{1b}$$

Thus, the results in terms of the SIFs can be obtained by considering the latter problem, which is analyzed easily by means of the hybrid displacement discontinuity method proposed recently by Yan [7].

* Corresponding author.

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3. Brief description of the hybrid displacement discontinuity method

In this section, the hybrid displacement discontinuity method presented by Yan [7] is described briefly. It consists of the constant displacement discontinuity element presented by Crouch and Starfield [8] and the crack-tip displacement discontinuity elements.

3.1. Constant displacement discontinuity method

The displacement discontinuity D_i is defined as the difference in displacement between the two sides of the segment [8] (see Fig. 2):

$$\begin{aligned} D_x &= u_x(x,0_-) - u_x(x,0_+) \\ D_y &= u_y(x,0_-) - u_y(x,0_+) \end{aligned} \tag{2}$$

The solution to the subject problem is given by Crouch and Starfield [8]. The displacements and stresses can be written as

$$\begin{aligned} u_x &= D_x[2(1-\nu)F_3(x,y) - yF_5(x,y)] + D_y[-(1-2\nu)F_2(x,y) - yF_4(x,y)], \\ u_y &= D_x[(1-2\nu)F_2(x,y) - yF_4(x,y)] + D_y[2(1-\nu)F_3(x,y) - yF_5(x,y)], \end{aligned} \tag{3}$$

and

$$\begin{aligned} \sigma_{xx} &= 2GD_x[2F_4(x,y) + yF_6(x,y)] + 2GD_y[-F_5(x,y) + yF_7(x,y)], \\ \sigma_{yy} &= 2GD_x[-yF_6(x,y)] + 2GD_y[-F_5(x,y) - yF_7(x,y)], \\ \sigma_{xy} &= 2GD_x[-F_5(x,y) + yF_7(x,y)] + 2GD_y[-yF_6(x,y)]. \end{aligned} \tag{4}$$

G and ν in these equations are shear modulus and Poisson's ratio, respectively. Functions F_2 through F_7 are described in Ref. [8]. Eqs. (3) and (4) are used by Crouch and Starfield [8] to set up a constant displacement discontinuity method.

3.2. Crack-tip displacement discontinuity elements

By using Eqs. (3) and (4), recently, Yan [7] presented crack-tip displacement discontinuity elements, which can be classified as the left and the right crack-tip displacement discontinuity elements to deal with crack problems in general plane elasticity. The following gives basic formulas of the left crack-tip displacement discontinuity element.

For the left crack-tip displacement discontinuity element (see Fig. 3), its displacement discontinuity functions are chosen as

$$D_x = H_s\left(\frac{a_{tip}+\xi}{a_{tip}}\right)^{\frac{1}{2}}, \quad D_y = H_n\left(\frac{a_{tip}+\xi}{a_{tip}}\right)^{\frac{1}{2}}. \tag{5}$$

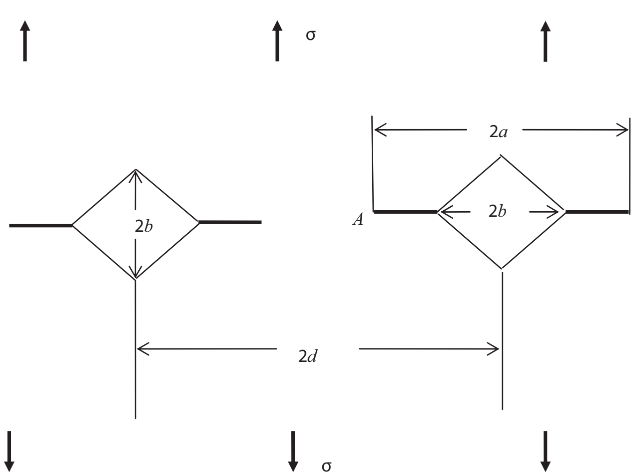

Fig. 1. Schematic of the interactions of two collinear square-hole cracks in an infinite plate in tension.

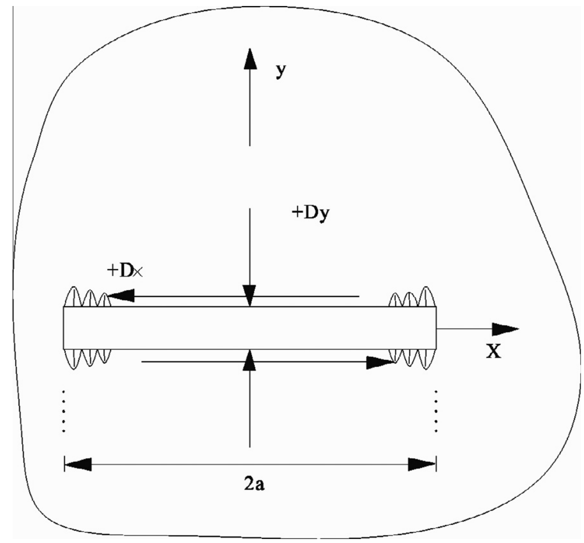

Fig. 2. Schematic of constant displacement discontinuity components D_x and D_y.

where H_s and H_n are the tangential and normal displacement discontinuity quantities at the center of the element, respectively, a_{tip} is a half length of crack-tip element. Here, it is noted that the element has the same unknowns as the two-dimensional constant displacement discontinuity element. But it can be seen that the displacement discontinuity functions defined in (5) can model the displacement fields around the crack tip. The stress field determined by the displacement discontinuity functions (5) possesses $r^{-1/2}$ singularity around the crack tip.

Based on Eqs. (3) and (4), the displacements and stresses at a point (x, y) due to the left crack-tip displacement discontinuity element can be obtained,

$$\begin{aligned} u_x &= H_s[2(1-\nu)B_3(x,y) - yB_5(x,y)] + H_n[-(1-2\nu)B_2(x,y) - yB_4(x,y)], \\ u_y &= H_s[(1-2\nu)B_2(x,y) - yB_4(x,y)] + H_n[2(1-\nu)B_3(x,y) - yB_5(x,y)], \end{aligned} \tag{6}$$

and

$$\begin{aligned} \sigma_{xx} &= 2GH_s[2B_4(x,y) + yB_6(x,y)] + 2GH_n[-B_5(x,y) + yB_7(x,y)], \\ \sigma_{yy} &= 2GH_s[-yB_6(x,y)] + 2GH_n[-B_5(x,y) - yB_7(x,y)], \\ \sigma_{xy} &= 2GH_s[-B_5(x,y) + yB_7(x,y)] + 2GH_n[-yB_6(x,y)], \end{aligned} \tag{7}$$

where functions B_2 through B_7 are described in Ref. [7].

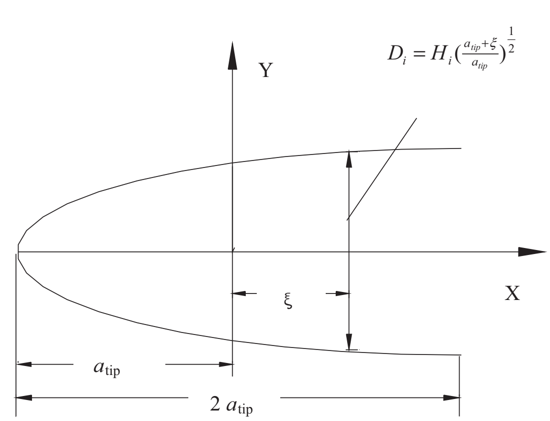

Fig. 3. Schematic of the left crack tip displacement discontinuity element.