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A local mesh replacement method for modeling near-interfacial crack growth in 2D composite structures

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ABSTRACT

An extended-finite-element-based method is proposed to accommodate the arbitrary motion of a crack in a general two-dimensional domain containing different kinds of material interfaces. To obtain the accurate stress intensity factors (SIFs) when the crack tip approaches the interface, the interpolation method in the vicinity of the crack tip employed in the extended finite element method (XFEM) is replaced with one that is derived from a moving mesh patch. Mesh configurations in this patch are the same as that adopted in the finite element method (FEM) for crack problems. The boundary of the patch is required to be coincident with background-mesh element edges and only the patch mesh works during the computation. As a result, the major advantages of the XFEM for modeling crack growth are preserved. The simulations are accomplished using a new domain expression of an interaction integral for evaluating stress intensity factors, and the maximum hoop stress criterion for crack-growth direction prediction. Several numerical examples are presented to prove the capability and practicability of the proposed technique and the program.

1. Introduction

The modeling of crack growth is a problem of great importance in the simulation of failure processes of engineering materials. Therefore, many techniques have been developed to take into account the discontinuity produced by a crack. Finite element method (FEM), which is one of the most powerful computational tools, has been widely used in these areas. One of the well-known methods is the re-meshing technique which adopts a discrete crack model for crack growth under quasi-static loading [1–3]. In addition, Nishioka et al. [4] developed a moving finite element method based on a mapping technique for dynamic fracture path prediction. In classical finite element analyses, it is necessary to re-mesh before assigning a crack increment because the element edges should be coincident with the crack path. For this issue to be overcome, several methods have been developed. Rashid [5] introduced an arbitrary local mesh refinement, where a static background mesh covers the whole structure and a moving patch mesh surrounds the crack tip. Only the patch mesh works in the region where it exists. Although the simulation of crack growth is facilitated by this method, the crack is still restricted to grow between elements. Alternatively, a variety of methods have been developed on the basis of the partition of unity method (PUM) proposed by Babuška and Melenk [6]. Among these methods, the partition of unity FEM, the generalized FEM and the extended FEM (XFEM) were elaborated by Fries and Belytschko [7]. The XFEM may be the most suitable choice for the analysis of structures with complex geometry since it allows the discontinuities, such as cracks, holes, inclusions to be independent of the mesh [8–10]. This advance has provided a robust and accurate computational tool for modeling discontinuities and their evolvement [11–14]. For more information about the XFEM, one can refer to the review paper written by Abdelaziz and Hamouine [15].

To the best of the author's knowledge, very few studies have been performed concerning the interaction between crack and material interface using the XFEM. Nielsen et al. [16] applied the XFEM to model crack growth near inclusions. A procedure to handle different propagation rates at different crack tips is presented. However, during the simulation, the distance between the crack and the inclusion is relatively large and the accuracy of the fracture parameters at the crack tips has not been discussed. The interaction between a crack and an inclusion in particle reinforced composite was numerically studied by Natarajan [17] using the XFEM. In order to obtain the accurate stress intensity factors when the crack tip approaches the interface aroused by the inclusion, a

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much refined mesh is required at the key region. As a result, the process of the simulation of crack growth near inclusions could become very cumbersome. Yan and Park [18] simulated the near-interfacial crack growth in a metal–ceramic layered structure by the XFEM and they employed the Heaviside functions only. Although there is no need to re-mesh during the modeling of crack growth, much refined mesh also should be placed near the interface and the crack tip. A program based on the XFEM was developed by Zhuang and Cheng [19] for the modeling of sub-interfacial crack growth in bi-materials. The influence of material interface and loading condition on crack growth trajectories were discussed. More recently, Jiang et al. [20] developed the XFEM for the computation of dynamic stress intensity factors (DSIFs) in the structures containing multiple discontinuities.

In a word, the simulations mentioned above have not considered the accuracy and the efficiency of the numerical method at the same time for the modeling of crack growth near interfaces. To deal with these problems, a local mesh replacement method based on the XFEM is developed in this paper. And then, a new domain expression of an interaction integral is employed for the calculation of fracture parameters at the crack tips. In particular, there is no need to avoid the interface using the interaction integral. The maximum hoop stress criterion is adopted to predict the crack growth direction. To validate the precision and the practicability of the proposed numerical technique, the simulation of a near-interfacial crack growth in a layered structure is presented first and the numerical results are compared with the available experimental results. Finally, the crack growth behavior in particle reinforced composite materials is investigated.

2. A local mesh replacement method based on the XFEM

The extended finite element method (XFEM) was developed by Belytschko and Black [8] and Moës et al. [9]. It is based on the concept of partition of unity given by Babuška and Melenk [6], who presenting a means to embed local solutions of boundary-value problems into the finite element approximation. Therefore, the XFEM allows the discontinuous boundaries, such as cracks or material interfaces, to be independent of the mesh. A notable improvement and progress of the XFEM is that there is no need for any re-meshing strategy in crack-growth modeling. However, in the XFEM for discontinuous problems, the corresponding analytical results are pre-requisite. If the analytical solutions are difficult to obtain or are very complex themselves, the application of the method will be impeded. Moreover, if the crack tip approaches the material interface, special manipulations should be conducted to ensure the accurate extraction of the fracture parameters at the crack tips using the XFEM. For these reasons, Yu et al. [21,22] proposed a novel method based on the XFEM and we provide only the key points herein. In order to describe the discontinuous characteristic, the following signed distance function introduced by Belytschko and Black [8] is adopted

$$f_\alpha(\mathbf{x}) = \min_{x^*\in\Gamma_\alpha}\|\mathbf{x}-\mathbf{x}^*\|sign(\mathbf{n}^+\cdot(\mathbf{x}^*-\mathbf{x})) \tag{1}$$

where $\mathbf{x}$ is a point in the domain Ω as shown in Fig. 1; $\mathbf{x}^*$ is a point on the discontinuous surface Γ_α, where $\alpha = c$ for the crack surface and $\alpha = p$ for the particle boundary; displacements are prescribed on Γ_u, tractions are prescribed on Γ_t and all the Γ_c are assumed to be a traction free surface; $\mathbf{n}^+$ is an unit outward normal vector to the surface. According to the signed distance function, we adopt the approximation of the displacement $\mathbf{u}(\mathbf{x})$ as

$$\begin{aligned}\mathbf{u}^h(\mathbf{x}) &= \sum_{I\in D_0}N_I(\mathbf{x})\mathbf{u}_I + \sum_{J\in D_1}N_J(\mathbf{x})\mathbf{b}_J\varphi_J(\mathbf{x}) + \sum_{K\in D_2}N_K(\mathbf{x})\mathbf{c}_K\psi_K(\mathbf{x})\\ \varphi_J(\mathbf{x}) &= |f_\alpha(\mathbf{x})| - |f_\alpha(\mathbf{x}_J)|,\;\; \psi_K(\mathbf{x}) = H(f_\alpha(\mathbf{x})) - H(f_\alpha(\mathbf{x}_K))\end{aligned} \tag{2}$$

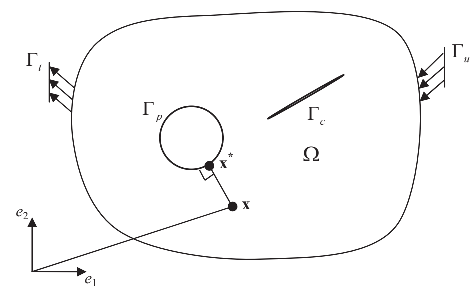

Fig. 1. Illustration of different discontinuous interfaces in a domain Ω.

Here, $N_I(\mathbf{x})$ is the standard finite element shape function; $\varphi_J(\mathbf{x})$ and $\psi_K(\mathbf{x})$ are the shifted enrichment functions for material interface and crack, respectively; $\mathbf{u}_I$ is the nodal displacement; As shown in Fig. 2(a), D_0 is the set of all nodes in the mesh; D_1 and D_2 are the sets of nodes enriched with $\varphi_J(\mathbf{x})$ and $\psi_K(\mathbf{x})$, respectively; $\mathbf{b}_J$ and $\mathbf{c}_K$ are the additional degrees of freedom for the nodes in D_1 and D_2, respectively; To improve the efficiency, of course, there are other similar choices of the enriched functions [23,24]. In this method, only the enrichment functions for material interface and crack surface are adopted. In order to improve the numerical precision, the mesh around the crack tip is refined as shown in Fig. 2(a). The degenerate elements are employed around the crack-tip just like the idea used in the traditional finite element method. Thus, this method can easily be applied to the problems in which the analytical expressions of crack-tip fields are difficult to obtain or are complex themselves. Yu et al. [21,22] has demonstrated the excellent accuracy and convergence of this numerical technique by resolving several benchmark problems in fracture mechanics of non-homogeneous materials. Here, we try to develop this method to make it suitable for crack-growth modeling in the materials containing different kinds of interfaces.

First, the location of the refined mesh are directly associated with the location of the crack-tip. Then, the local domain occupied by elements surround the crack tip (in this paper, the shaded 9 elements as shown in Fig. 2(b) are adopted) is refined as mentioned above. Finally, we lay the refined mesh over the original structured mesh. They share the same nodes with each other on the boundary of the refined domain. The initial values of stiffness of the 9 original quadrilateral elements are assigned to zero, and will not be computed any more. During the simulation, the refined mesh moves together with the crack tip just like a patch, as shown in Fig. 2(a and b). These manipulations insure that only the patch mesh works in the refined domain and the structured mesh is not destroyed during the crack-growth modeling. With this method, only the patch region should be refined and there is no need to ensure the crack surface and the material interfaces confirm with the mesh boundaries. Up to now, we can find that this method not only maintains the good idea of the XFEM, but also can be used in more general situations. The variational formulation of the boundary-value problem of a fundamental static equation and the corresponding discretized form are the same as that adopted in the XFEM [8].

3. The interaction integral

In the present section, the interaction integral when the integral domain contains material interface will be introduced. Our attention is restricted to plane problems, the material is limited to linear-elastic and small strain kinematics is assumed.

The interaction integral is derived from the J-integral for two admissible states (actual and auxiliary fields). We denote the stress σ_{ij}, strain ε_{ij} and displacement u_i fields stemming from the solution