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Isogeometric analysis of stress intensity factors for curved crack problems

Myung-Jin Choi, Seonho Cho *

National Creative Research Initiatives (NCRI) Center for Isogeometric Optimal Design, Department of Naval Architecture and Ocean Engineering, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 151-744, Republic of Korea

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ABSTRACT

An isogeometric analysis method is developed for the stress intensity factors in curved crack problems. In the isogeometric approach, NURBS (Non-Uniform Rational B-Spline) basis functions in CAD system is directly utilized in the response analysis, which enables the seamless incorporation of the higher continuity and the exact geometry such as curvature and normal vector into the computational framework. In mixed-mode curved crack problems, the precise evaluation of crack-face integral is essential to compute the precise stress intensity factors, especially for the path-independency of interaction integral. The CAD-based exact representation of tangential and normal vectors facilitates to exactly define a local coordinate system at the crack-tip, and accurately evaluate the crack-face integral. Compared with the standard finite element approach, a higher continuity of stress and strain fields are expected in the interaction integral domain. Various numerical examples of curved crack problems are presented to demonstrate the accuracy of developed isogeometric method through the comparison of both exact and finite element solutions.

1. Introduction

One of the most challenging issues in structural mechanics is structural failure due to fatigue and fracture. The identification of crack initiation and growth is crucial for the evaluation of structural safety. Many researches based on linear elastic fracture mechanics have been performed to predict the behavior of the crack. Recently, Nam et al. [1] apply the control of crack initiation, propagation, and termination to microscopic pattern generation. The controlling method could overcome the problem of deteriorated resolution of patterns due to electron diffraction in the existing lithography method by an electron beam, together with a significant reduction of operation time for pattern generation. Through numerous experiments, this research shows the feasibility of micro-notch design to obtain a desired crack path. However, for the precise control of the crack formation, it is necessary to predict the initiation and stability of the crack growth using precise stress intensity factors (SIF). The derivative of SIF is helpful to predict the direction of crack growth together with the instability of a fracture or life cycle. As the first order derivative of potential energy, Feijoo et al. [2] discussed a finite element based shape sensitivity analysis for the energy release rate (G) in cracked structures.

In analyzing crack propagation, even though standard finite element analysis (FEA) is suited for the approximation of smooth solutions and thus widely used in engineering practice, it often encounters non-smooth problems which might result in computational difficulties. In particular, in the analysis of crack propagation problems, the FEA requires a tedious re-meshing process and the alignment of crack lines with the edge of finite elements. To overcome these difficulties, Belytschko and Black [3] proposed an extended finite element method (XFEM), which provides flexibility and versatility in the modeling of discontinuity that does not require the alignment of crack lines as well as the re-meshing of finite elements. The solution space is locally enriched in the space of FEA approximation and its discontinuity is implicitly embedded inside the element. Moës et al. [4] incorporated the discontinuous displacement field using a Heaviside function at the crack surface away from the crack tip and implicitly expressed the path of crack growth combined with the level set method. Stazi et al. [5] developed a method for incorporating the step function and the near-field crack-tip enrichment by discontinuous partitions of unity in higher order elements.

The interaction integral (M-integral) formulation derived from the J-integral which considers a superposition of actual and auxiliary field [6] is known to be the most accurate and efficient method

* Corresponding author.
E-mail address: secho@snu.ac.kr (S. Cho).

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Nomenclature

$X_I, (I = 1, 2)$ or X, Y global rectangular Cartesian coordinates
$x_i, (i = 1, 2)$ or x, y local rectangular Cartesian coordinates at crack-tip
$\mathbf{X}^c$ position of crack-tip in global Cartesian coordinates
$n_i, (i = 1, 2)$ normal vector in local Cartesian coordinate at crack-tip
Γ_c^+, Γ_c^- upper and lower crack-faces

for calculating SIFs under mixed-mode loading conditions. Gosz and Moran [7] adopted the interaction integral method to study 3D non-planar cracks in homogeneous materials. Using the interaction integral formulation, Walters et al. [8] computed the SIF in mixed mode for 3D curved crack problems subjected to surface tractions. They point out that for the evaluation of the response in the auxiliary field, the local coordinates of a crack-tip should be highly accurate but could be inaccurate due to geometric approximation errors if linear finite elements are used. To overcome the aforementioned difficulty, they employ a local orthogonal coordinate system but require a highly refined mesh for precise computation. Yu et al. [9] extended the interaction integral method to solve the fracture problems in 3D nonhomogeneous materials with arbitrary interfaces in the integral domain. Nevertheless, the geometric approximation in numerical methods has always been an important and persisting issue to overcome.

Recently, taking advantage of the NURBS (Non-Uniform Rational B-Spline) basis functions, Hughes et al. [10] developed an isogeometric analysis (IGA) method that has many advantages over the standard FEA in various engineering applications such as structural vibrations, fluid–structure interactions, and turbulent flow simulations. The geometric approximation which is inherent in the finite element mesh could lead to accuracy problems in response. The piecewise linear approximations of geometry turned out to be the root cause. The major feature of the isogeometric method is to employ the same NURBS basis functions as used in the CAD systems, which is a significant advantage of exact representation of geometry. The isogeometric analysis method has many features in common with the FEA; it invokes the isoparametric concept where the dependent variables and the geometry share the same basis functions. Also, the method has some features in common with the meshfree methods; it is not interpolatory. The isogeometric method is rapidly extending its applications in areas such as shell analysis [11], shape sensitivity analysis and optimization [12], and adaptive shape optimization enhanced by T-splines [13]. The isogeometric method has a major feature such as the CAD based parameterization of field variables in an isoparametric manner and thus requires no further communication with the CAD systems during refinement processes.

Verhoosel et al. [14] employ the isogeometric method having higher order continuity for the approximation of higher order gradient damage, which overcomes the limitation of conventional FEA having C^0 continuity. Verhoosel et al. [15] use the NURBS basis function for the discretization of cohesive zone formulation, and represents the discontinuity through the insertion of knots. Also, the evolution of T-spline mesh is utilized to represent the growth of crack. To improve the solution accuracy in the isogeometric method, De Luycker et al. [16] employ the enrichment at control points to represent the discontinuity and singularity at the crack-tip. Only the problem of mode-I straight crack is considered so that there is a still limitation on the solution accuracy in curved crack problems due to geometry approximation even though a sub-triangle technique is used for the partial numerical integration of elements. Ghorashi et al. [17] also uses the enrichment and sub-triangle technique for partial numerical integration in the isogeometric analysis, and tried to solve a mixed mode straight crack. For general curved crack problems, the geometry is still approximated by straight lines and thus the accuracy of stress intensity factor is not thoroughly verified. In this paper, the curved geometry of the cracks is exactly represented by taking advantage of the isogeometric approach.

This paper is organized as follows: *in* Section 2, we briefly summarize the NURBS basis function and explain how to represent the discontinuity for cracks by a CAD model. *In* Section 3, using an interaction integral formulation, the IGA method for SIFs is presented, taking advantage of exact geometry to define a local coordinate system at a crack-tip in the isogeometric approach. *In* Section 4, several numerical examples are presented to demonstrate the advantage of the developed IGA method for the curved crack problems. The IGA results are verified with both finite element and exact solutions. Finally, the importance of the developed IGA method and its superior points especially for the curved crack problems are discussed in the concluding remarks.

2. NURBS basis function

2.1. B-spline basis function

In the IGA, the solution space is represented in terms of the same basis functions used for representing the geometry. The IGA method has several advantages over the conventional FEA such as exact geometry and simple refinements which are the results of the use of NURBS basis functions based on B-splines. Consider a knot vector $\boldsymbol{\xi}$ in one dimensional space, which consists of knots ξ_i in a parametric space.

$$\boldsymbol{\xi} = \left\{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\right\}, \tag{1}$$

where p and n are the order of the basis function and the number of control points, respectively. B-spline basis functions are recursively defined as

$$N_i^0(\xi) = \begin{cases} 1 & \text{if } \xi_i \leqslant \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}, (p = 0) \tag{2}$$

and

$$N_i^p(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_i^{p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p-1} - \xi_{i+1}} N_{i+1}^{p-1}(\xi), (p = 1, 2, 3, \ldots). \tag{3}$$

Using the B-spline basis function $N_i^p(\xi)$ and the corresponding weight w_i, a NURBS basis function $R_i^p(\xi)$ is defined as

$$R_i^p(\xi) = \frac{N_i^p(\xi) w_i}{\sum_{j=1}^n N_j^p(\xi) w_j}. \tag{4}$$

Generally, an isogeometric approach using higher order basis functions offers higher regularity than the conventional FEA. The NURBS possesses the following desirable properties as a basis function:

(1) $\sum_{i=1}^l R_i^p(\xi) = 1$ (partition of unity).
(2) R_i^p is included in the interval $[\xi_i, \xi_{i+p+1}]$ (compact support).
(3) $R_i^p(\xi) \geqslant 0$ (non-negativity).