Theoretical and Applied Fracture Mechanics 75 (2015) 113–123

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Stress intensity factors for internal surface cracks in autofrettaged functionally graded thick cylinders using weight function method

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A R T I C L E I N F O

Article history:
Available online 18 December 2014

Keywords:
Functionally graded cylinder
Weight function method
Stress intensity factor
Autofrettage
Residual stress

A B S T R A C T

Stress intensity factors for internal surface cracks in autofrettaged functionally graded cylinders have been studied. It is assumed that the mechanical properties change with a power law in radial direction of cylinder. Effects of autofrettage pressure, volume fraction, crack dimensions and cylinder thickness on the stress intensity factors in deepest and surface points of cracks were determined by using the weight function method. Results show that the changes in volume fractions of ingredients have more effects than the other parameters.

1. Introduction

Autofrettage is a process for inducing compressive residual stress in internal parts of the thick walled cylinders by using the high internal pressure. The pressure is large enough to cause material yielding within the wall. After removing the applied pressure, partially yielding creates the compressive residual stress at the inner part while the outer part has a tensile residual stress. The compressive residual stress can reduces the rate of fatigue crack growth in the materials.

Functionally graded materials (FGMs) are special composites where the volume fraction of constituent materials vary gradually and produce non-uniform microstructure with continuously graded macro-properties. There is no discontinuity in the thermal, mechanical and other properties. Change in type and volume fraction of ingredients can optimize the FGM to achieve high performance and material efficiency. There are no distinct internal boundaries inside the material and thus minimal interfacial stress concentrations are developed. These materials are used extensively in applications where the operating conditions are usually severe. For instance, due to simultaneous thermal and mechanical loadings, stress concentrations are developed, which can lead to cracking in ceramic–metal composites. In order to avoid this, the FGMs with gradual variation in the properties are being used. These graded materials can be used also in wear resistant linings for handling heavy abrasive ore particles, rocket heat shields, heat exchanger tubes, thermoelectric generators, heat-engine components and many applications which require minimization of thermo-mechanical mismatch in metallic-ceramic bonding.

Determination of parameters of cracks in the autofrettaged functionally graded thick cylinders (FGCs) can be useful in study of the behavior of these materials in critical conditions. Stress intensity factors (SIFs) can be used to predict the fracture behavior and optimize the design of the FGM composites. Because of graded behavior of FGMs, the SIFs were determined usually in critical points such as deepest and surface points of the crack front.

The fatigue and fracture behavior of a pressure vessel is significantly affected by residual stress. Using the autofrettage process for generation of residual stress in FGMs has been investigated in the literature. Jahromi et al. [1] studied the residual compressive stresses induced in an autofrettaged pressure vessel made of the FGM by developing the variable material property (VMP) method for materials with varying elastic and plastic properties versus position. They showed that the reinforcement of an autofrettaged metallic cylinder by ceramic, with an increasing ceramic volume fraction from inner to outer radius, increases the compressive residual stresses at the inner section. Distribution of the residual stresses induced by autofrettage process in layered and functionally graded composite vessels has been achieved [2]. Calculations were carried out using an extension VMP method and validated in several cases using finite element calculations. This study showed that the induced residual stress at the inner surface of composite vessels could reach much higher values compared to a metal vessel counterpart, depending on the properties of the composite constituents.

Applying severe stress gradients on the FGMs which induced by thermal or mechanical loadings can lead to the formation of some surface defects such as cracks. These imperfections, in turn, can seriously endanger the structural integrity [3]. Therefore, it is

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http://dx.doi.org/10.1016/j.tafmec.2014.11.004
0167-8442/

necessary to study the fracture behaviors of the FGMs in various conditions and under composition of different loadings like residual and/or thermal stresses. There are several studies where performed on the cracked FGMs. Some researches were done on calculating the SIFs using the concept of J-integral and interaction integral (which involves both actual and auxiliary fields in J-integral calculation) for various loadings on the FGMs [3–10]. Also energy methods such as strain energy density (SED) were employed to determine the failure and crack growth direction in FGMs [11,12] Other investigations were performed using the displacement fields around crack tip or crack front (calculation of crack opening displacement (COD)) [13–20]. Some researchers calculated the SIFs in two dimensional conditions by using the limiting proper expression of near field stresses (multiply of stress in radical of difference between crack tip location and parallel coordinate to crack line) [21] which be used for homogeneous materials.

Fett et al. [22] studied the applicability of weight function (WF) method to predict the SIFs in the FGMs. They concluded that, if Young's modulus at the crack tip is used in relations, the Rice equation among the displacements and the weight function is valid also for graded materials.

There are some studies on the evaluation of the SIFs due to the residual stress by WF method. Shen et al. [23] used this method for calculating the residual SIFs for axial cracks in an autofrettaged metal cylinder. Also, Bao et al. [24] evaluated the SIFs due to welding residual stresses using FEM and WF methods. They recommended that measured residual stresses should be firstly processed, e.g. smoothed, fitted and balanced, before conducting analysis to obtain more accurate results for the residual SIFs.

In this paper, we used a proper subroutine (UMAT) in the FEM (abaqus) software to calculate the residual stress distribution due to autofrettage on a functionally graded cylinder. By using the weight function method, the stress intensity factors in deepest and surface points of internal semi-elliptical cracks were obtained. The cracks are axially oriented. Effects of several variables such as crack geometries, volume fractions of FGM components, autofrettage and applied pressures and thickness of the cylinder on the residual SIFs were studied.

2. Micromechanics of FGM

The material properties of the FGM, such as Young's modulus, often are evaluated from properties of the constituent materials using micromechanics models. Based on the rule of mixtures, a simple model, which is identified as "Tamura–Tomota–Ozawa" (TTO) model [25], is proposed to describe the elastic–plastic stress–strain curves of graded materials [26]. This model relates the uniaxial stress, σ, and strain, ε, of the FGM to the corresponding uniaxial stresses and strains of the two constituent materials by:

$$\sigma = V_m\sigma_m + V_c\sigma_c,\ \ \varepsilon = V_m\varepsilon_m + V_c\varepsilon_c,\ \ V_m + V_c = 1.0 \tag{1}$$

where $V = V(r)$ denotes the volume fraction of the constituent material which varied with radial position, r, and subscripts m and c are used for ductile (metal) part and brittle (ceramic) part, respectively. In the TTO model, an additional parameter, q, is defined to represent the ratio of stress-to-strain transfer, as follows [27]:

$$q = \frac{\sigma_c - \sigma_m}{\varepsilon_c - \varepsilon_m}, \quad 0 < q < \infty \tag{2}$$

In which $q = 0$ and $q \to \infty$ correspond to property averaging with equal stress and equal strain, respectively. Based on the linear Hooke's law, Young's modulus, E, of the FGM may be obtained from equations 1 and 2, as:

$$E = \frac{V_mE_m + mV_cE_c}{V_m + mV_c}, \quad m = \frac{q + E_m}{q + E_c} \tag{3}$$

Whereas, brittle material does not yield, the initial yield stress (S_Y) of the FGM is as follows [26,27]:

$$S_Y = S_{Ym}(V_m + mV_cE_c/E_m) \tag{4}$$

By assuming the bilinear relation between the stress and strain for FGM as depicted in Fig 1, its tangent modulus, H, can be written as:

$$H = \frac{V_cE_c + hV_mH_m}{V_c + hV_m}, \quad h = \frac{q + E_c}{q + H_m} \tag{5}$$

where H_m is the tangent modulus of metal.

Schematic of mentioned variables is shown in Fig 1.

Because of variation of the volume fraction versus radial position in the cylinder, the mechanical properties also change in the wall.

2.1. Governing equations

It is assumed that the FGM is locally isotropic and yields according to the von Mises criterion. For a cylindrical pressure vessel, the material properties of a point located at a distance $x = r - R_i$ from internal radius (R_i) can be represented by an elastic modulus, $E(x)$, and constant Poisson's ratio, ν and a curve that represents the plastic behavior. This curve can be written in the following form:

$$\partial\overline{\sigma}_Y/\partial\overline{\varepsilon}^p = H(x, \overline{\varepsilon}^p) \tag{6}$$

where $\overline{\sigma}_Y$ and $\overline{\varepsilon}^p$ are the yield stress and equivalent plastic strain, respectively. If $H(x, \overline{\varepsilon}^p)$ do not depend on the $\overline{\varepsilon}^p$ (i.e. $\overline{\sigma}_Y$ versus $\overline{\varepsilon}^p$ is a line as depicted in Fig 1) then we can write:

$$\overline{\sigma}_Y = S_Y(x) + H(x)(\overline{\varepsilon}^p) = S_Y(x) + H(x)(\overline{\varepsilon} - S_Y(x)/E(x)) \tag{7}$$

where $\overline{\varepsilon}$ is the equivalent total strain.

The components of strain tensor, ε_{ij}, is the summation of the elastic part, ε^e_{ij}, and plastic part, ε^p_{ij} (i.e. $\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^p_{ij}$ or in the incremental form $d\varepsilon_{ij} = d\varepsilon^e_{ij} + d\varepsilon^p_{ij}$). The elastic part is given as,

$$\varepsilon^e_{ij} = \frac{1+\nu}{E(x)}\sigma_{ij} - \frac{\nu}{E(x)}\sigma_{kk}\delta_{ij} \tag{8}$$

where δ_{ij} is the Kronecker symbol. By using the definition of deviatoric stress tensor $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3$ and deviatoric elastic strain tensor $e_{ij} = \varepsilon^e_{ij} - \delta_{ij}\varepsilon^e_{kk}/3$, Eq (8) can be converted to the following equation:

$$s_{ij} = 2Ge_{ij} \tag{9}$$

where $G(x) = E(x)/2(1+\nu)$ is the shear moduli of FGM.

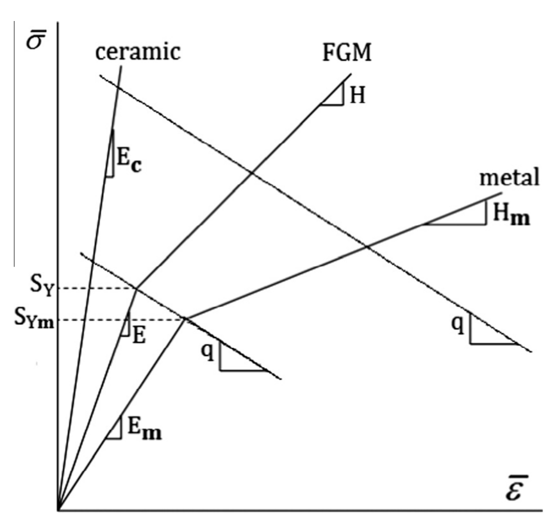

Fig. 1. Schematic of uniaxial stress–strain curve of the FGM (TTO model [25]).