Theoretical and Applied Fracture Mechanics 59 (2012) 21–28

Contents lists available at SciVerse ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Local strain energy density applied to martensitic steel plates weakened by U-notches under mixed mode loading

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ARTICLE INFO

Article history:
Available online 9 May 2012

Keywords:
Brittle fracture
Mixed mode loading
Strain energy density (SED)
Fracture load

ABSTRACT

The averaged value of the strain-energy density over a well-defined volume is used to assess the static strength of U-notched specimens under mixed mode loading (I + II). The volume is centered in relation to the maximum principal stress present on the notch edge, by rigidly rotating the crescent-shaped volume already used in the literature to analyze U- and V-shaped notches under mode I loading. In total 96 new tests have been carried out on specimens made of structural steel with a martensite phase weakened by U-notches. The notch root radius varies from 0.2 to 2.0 mm while the notch depth is equal to 5, 7.5, 10 and 15 mm, respectively. In addition, for the lowest value of the notch depth (5 mm), some data from cracked specimens are summarized in the paper.

Good agreement was found between experimental results and theoretical assessment based on the averaged strain energy density criterion under mixed mode loading.

1. Introduction

Several criteria have been proposed to predict fracture loads of components with pointed (sharp) [1–7] and blunt V-notches [8–22], subjected to mode I loading. Dealing with fracture assessment of sharp and rounded V-notches the main problem is to adapt the same criterion to different cases independently of the notch radius. One of the criteria that permits to overcome this problem, unifying all the notch geometries and acuities, is based on the averaged value of the local strain energy density (SED). This criterion has been used to summarize static data from sharp and blunt notches in materials exhibiting a brittle or quasi-brittle behavior [23–26], high cycle fatigue data from welded joints under prevalent mode I loading [27,28], fatigue data from shrink-fit couplings [29] and also data from static tests under mixed mode loading [30–33]. A wide review of the approach is carried out in Ref. [34].

The problem of brittle failure from blunt notches loaded under mixed mode is more complex than under mode I loading and experimental data are very scarce in particular dealing with metallic materials weakened by rounded notches.

Under mixed mode loading, several fracture criteria have been proposed in the recent and past literature [35–47]. Under mode I loading, fracture systematically initiates from the notch tip located on the notch bisector line; this is the point where the principal stress reaches its maximum, as well as the strain energy density. On the other hand the point where fracture initiates in mixed mode varies from case to case, because it depends on the geometry of the notch and mode mixity due to the scheme of application of the external load.

The weakness of brittle materials can be generally attributed to the non-uniform distribution of internal and/or surface flaws microcracks resulting in a non-continuum surface. These flaws are inherent in many engineering materials and they also persist around the edge of a blunt notch. Hence, a failure criterion such as the maximum surface tangential stress (MTS) was unable to predict the load that may be safely applied to the material for high stress gradient fields [8]. A coherent theory of fracture that can consistently account for the fracture behavior of both sharp cracks and blunt notches was proposed in Refs. [40–47] by applying the strain energy density criterion to the bulk of the material near the notch surface. While the surface flaws may locally invalidate the continuum solution, the solution remains satisfactory in the bulk of the material [9].

The main aim of the present paper is to provide a new set of experimental data on fracture of U-notched steel samples under mixed mode loading. To achieve different values of mode mixity different test configurations have been used. Notches with different radii (from 0 to 2 mm) and different notch depths (from 5 to 15 mm) have been considered varying also the span length for the application of the load.

The new results may be helpful for researchers because enlarges the scarce available data. In fact, at the best of authors' knowledge, no data exist for steel specimens under mixed mode loading with the intensity of mode II comparable with that due to mode I.

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http://dx.doi.org/10.1016/j.tafmec.2012.05.003

In addition, a simple criterion based on strain energy density has been applied to this set of experimental results showing a sound capability for the fracture load assessment.

2. Strain-energy density over the control volume under mixed mode loading

The averaged strain-energy density criterion (SED) as reported in Ref. [23] states that brittle failure occurs when the mean value of the strain energy density over a control volume is equal to the critical energy for the un-notched material, W_c. The SED approach is based both on a precise definition of the control volume and the fact that the critical energy does not depend on the notch sharpness. Such a method was applied first to sharp, zero radius, V-notches and later extended to blunt U- and V-notches under mode I loading [24]. For a blunt V-notch under mode I loading, the volume assumes the crescent shape shown in Fig. 1, where R_c is the depth measured along the notch bisector line. The Cartesian coordinate origin is located on a certain distance r_o from the notch tip that depends both on the notch root radius (ρ) and the opening angle 2α, according to the following expression [48,49].

$$r_o = \rho\left(\frac{\pi - 2\alpha}{2\pi - 2\alpha}\right) \tag{1}$$

For U-notches r_o is simply equal to $\rho/2$, as for blunt-cracks [50]. If the fracture toughness K_{IC} is known, the expression for R_c [25] under plane strain conditions, respectively, becomes:

$$R_C = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_u}\right)^2 \quad \text{plane strain} \tag{2}$$

In Eq. (2) σ_u is the ultimate strength of the material and ν the Poisson's ratio.

The mean value of SED over the control volume Ω defined in Fig. 1 can be expressed in the following form, as discussed in Refs. [24,30,31]

$$\overline{W} = H(R_c/\rho, \nu)\frac{\pi\sigma_{max}^2}{4E} \tag{3}$$

where σ_{max} is the maximum elastic stress at the notch tip and H is a function of R_c/ρ and ν. Function H for some values of the ratio R_c/ρ and ν is listed in Table 1.

The critical volume in U-notched specimens under mode I loading conditions is centered in relation to the notch bisector line (Fig. 2a). Under mixed mode loading the critical volume is no longer centered on the notch tip, but rather on the point where the principal stress reaches its maximum value along the edge of the notch (Fig. 2b). It is assumed that the crescent shape volume rotates rigidly under mixed mode, with no change in shape and size [30,31]. This is the governing idea of the '*equivalent local mode I*' approach. In this paper, the same idea has been used for mixed mode loading of steel plates.

For *mixed mode* loading an equivalent expression for the averaged strain energy density has been proposed in Ref. [31]:

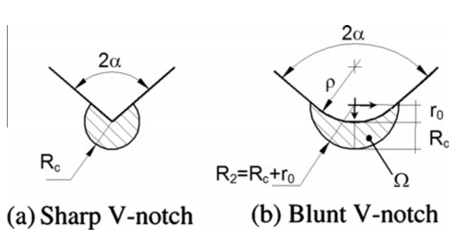

Fig. 1. Control volume for sharp V-notch (a); and blunt V-notch (b) under mode I loading; distance $r_o = \rho\ (\pi - 2\alpha)/(2\pi - 2\alpha)$ [24].

Table 1
Values of H parameter for U-notched specimens for some values of R_c/ρ and ν under plane strain conditions.

R_c/ρ	H		
	$\nu = 0.3$	$\nu = 0.35$	$\nu = 0.4$
0.01	0.5638	0.5432	0.5194
0.05	0.5086	0.4884	0.4652
0.1	0.4518	0.4322	0.4099
0.3	0.3069	0.2902	0.2713
0.5	0.2276	0.2135	0.1976

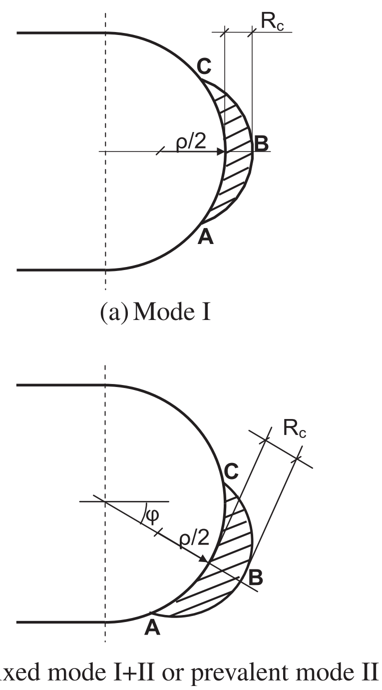

Fig. 2. Critical volume in U-notches; (a) under mode I loading; (b) under mixed mode (I + II) or under prevalent mode II loading.

$$\overline{W} = H^*(R_c/\rho, \nu)\frac{\pi(\sigma_{max}^*)^2}{4E} \tag{4}$$

where σ_{max}^* is the maximum value of the principal stress along the notch edge and H^* depends again on the normalized radius R_c/ρ, the Poisson's ratio, ν, and the loading conditions.

It was found that the difference between H and H^* is less than 1% [31]. Therefore, the following expression was proposed under mixed mode loading:

$$\overline{W} = H(R_c/\rho, \nu)\frac{\pi(\sigma_{max}^*)^2}{4E} \tag{5}$$

The geometry of the specimens considered in this paper is shown in Fig. 3. Also for the considered geometries, it is interesting to compute the value of H^* and compare it with H as made in Ref. [31].

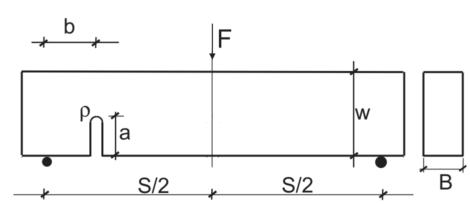

Fig. 3. The geometry of the specimen used in this paper.