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A model for reliability and confidence level in fatigue statistical calculation

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A B S T R A C T

A new kind of statistical data model which described the fatigue cracking growth with limited data was proposed, and the effects of the reliability and the confidence level to the fracture growth were considered. The one-sided allowance factor statistical analysis method was used to provide the prediction of the fatigue life with the confidence level and the reliability, and the effect factors were revised, which were closer to the lower limit of the matrix hundred rank values. It was found that this method gave much more accurate fatigue life prediction by analyzing the statistical data of the 7050 aluminum alloy before and after laser shock processing (LSP) and the new one-sided allowance coefficient saves more test samples in the same situation of precision. The revision coefficient would also save the experimental work load in the experiment is.

1. Introduction

Stress-based fatigue analysis has been favored in many engineering practices recently, and several valuable efforts have been made. The conventional fatigue reliability method bases on the survival probability (reliability) curve [1], namely the P–S–N curve, which is reliable only when the quantity of experimental data is big and uncertainty factors that affect the fatigue performance are exposed. The conventional method mentioned the concept of confidence level [1], but it has not yet developed the random reliability S–N curve category. Whilst tests on simple components could provide some confidence, increasing understanding and prediction accuracy, while it is necessary to test full size components in their in-service environment with a representative load regime [2–4]. Recently, one-dimensional loading was carried out to take into account the probabilistic effects on the fatigue behavior by giving a probabilistic stress–number of cycle's curves [5,6]. The weakest-link theory is the first approach treating the statistical effect in fatigue. An approach uses the Weibull model [7]. This model has been applied to bearing steel by Bomas et al. [8] and a cast iron to explain the statistical distribution on fatigue strength by Hild and Roux [9] and Chantier et al. [10] to explain the statistical distribution on fatigue strength. More recently, a combination of the concept of the weakest-link and a critical plane damage model based on a micro plasticity analysis to describe the distributions of the fatigue limit and the fatigue life under different loadings was analyzed by Morel [11]. Hence, an alternative approach is commonly adopted in order to evaluate the reliability in engineering design, experimental or numerical simulation and load histories are counted, as they are in deterministic-approaches [12–15]. Then the obtained counted cycles are statistically analyzed by means of specific procedures.

The coefficient of variation is caused to rise in a large scale, thus the request of the number of the smallest test sample is difficult to be satisfied in the short life area frequently, the insufficient data and the uncertainty factors that result to the measure results tend to lead a possible danger. So the concept of the one-sided allowance coefficient is needed to give lower confidence of matrix hundred rank values with the confidence level. Therefore, seeking for the fatigue reliability analysis method which considers situation of limited data with the important theory significance and the project application should be worthy to be explored. The object of this paper is to develop a methodology for stress-based reliability analysis taking into account of the scatter. The effects of reliability and the confidence level on fracture growth were considered, and the fatigue life with the confidence level and the reliability was given by using the one-sided allowance factor statistics analysis principle. The fatigue data model was optimized under the laser load by using the fracture mechanics theory. The one-sided allowance coefficient was revised, which was closer to the lower limit of the confidence of the matrix hundred rank value, and the usability and validity of the revision model were confirmed by analyzing the tentative data of 7050 aluminum alloy after LSP.

2. Analysis of statistics model

Usually, there should be enough test specimens so that the security fatigue life and the fatigue strength obtained from the

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fatigue experiment are requested to a certain confidence level. But to the actual components, there are only a few test specimens (5–11 numbers) as the result of the economic problems. If the number of test specimens cannot satisfy the request of the observed value, fatigue life with the confidence level γ and reliability p could be given by using the one-sided allowance factor k [16].

As the logarithm fatigue life follows the normal distribution, the logarithm security fatigue life x_p with the reliability p could be calculated as

$$x_p = \mu + u_p\sigma \tag{1}$$

Where to any assigned reliability level p, the value of u_p may be determined, which makes some individual logarithm fatigue life with p in the matrix become bigger than x_p. While as to the random variable function ε, the probability of the security life is smaller than the matrix's true value, which is expressed as the confidence level γ [16],

$$P(\bar{x} + k\beta s < \mu + u_p\sigma) = \gamma \tag{2}$$

As assumed that ε follows the normal distribution N, formula (1) could be changed into as follows,

$$\mu + u_p\sigma = E(\varepsilon) + u_\gamma\sqrt{Var(\varepsilon)} \tag{3}$$

According to the statistics theory, it could be defined by

$$E(\varepsilon) = E[\bar{x} + k\beta s] = E(\bar{x}) + kE(\beta s) = \mu + k\sigma \tag{4}$$

$$\begin{aligned} Var(\varepsilon) &= Var(\bar{x}) + k^2\beta^2 Var(s) \\ &= \frac{\sigma^2}{n} + k^2\beta^2\frac{\sigma^2}{n-1}\left\{n - 1 - 2\left[\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\right]^2\right\} \\ &= \frac{\sigma^2}{n} + k^2\left\{\frac{(n-1)\sigma^2}{2}\left[\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}\right]^2 - \sigma^2\right\} \end{aligned} \tag{5}$$

As $n = 5$, the formula (5) could be written as

$$Var(\varepsilon) = \sigma^2\left[\frac{1}{n} + \frac{k^2}{2(n-1)}\right] \tag{6}$$

So the formula (1) could be acquired as

$$\mu + u_p\sigma = \mu + k\sigma + u_\gamma\sigma\sqrt{\frac{1}{n} + \frac{k^2}{2(n-1)}} \tag{7a}$$

$$k = \frac{u_p - u_\gamma\sqrt{\frac{1}{n}\left[1 - \frac{u_\gamma^2}{2(n-1)}\right] + \frac{u_p^2}{2(n-1)}}}{1 - \frac{u_\gamma^2}{2(n-1)}} \tag{7b}$$

where k is the one-sided allowance factor, n is the number of the observed values, u_p and u_γ are partial volumes of normal related with the reliability and confidence respectively; and the values of u_γ under the confidence levels of 90% and 95% are 1.282 and 1.645, respectively [16]. The security logarithm life with the confidence level γ and reliability p could be expressed as

$$\hat{x}_p = \bar{x} + k\beta s \tag{8}$$

where u_p, u_γ, and n are all known, the factor k could be obtained by the formula (7b), and then security life with the confidence level γ and the reliability p could be obtained by the formula (8) as

$$\widehat{N}_p = \lg^{-1}\hat{x}_p \tag{9}$$

That is to say, some individual fatigue life with p at the confidence level of γ is bigger than $\widehat{N}_p$ in the matrix at least.

3. LSP and fatigue experiment

LSP is a fatigue enhancement surface treatment for metallic materials in which residual compressive stresses are mechanically produced into the surface [17–19]. LSP in a confined geometry was distinctly different from direct ablation because the coating layer was vaporized in the case of confined geometry whereas the work-piece itself was vaporized in the case of direct ablation. The collisions among the vaporized particles in the vaporized layer must be considered in the case of confined geometry, and the recoil and plasma pressures were of interests to calculate the pressure on the substrate surface, as shown in Fig. 1. 7050-T7451 aluminum alloy is used as the test specimen, which is prefabricated with cracks on its surface. The sizes of samples are shown in Fig. 2. The chemical composition of the sample and its material parameters are shown in Tables 1 and 2 separately. The fatigue test is completed on PLG-100C high-frequency fatigue testing machine. The stress fatigue lives of the test samples before and after LSP were compared. Fatigue experiments wuth the stress level from 60 Mpa to 140 MPa of the test samples after the processing were carried out. The experimental results showed that stress fatigue life of the test sample after LSP is 1–2 times of that of the non-shocked test sample. With regard to the non-through crack surface after LSP, the residual stress may witness significant variation under the instant LSP, which may further incur the variation of crack surface stress intensity factor along the crack front.

The experimental result was carried on data statistics processing, the stress-crack initiation life data was shown in Fig. 3. Fig. 4 gives the fitting effect of the one-sided allowance factor method to the tentative data with the conventional reliability of 0.95 and different confidence levels, it could be known from Fig. 4 that the fatigue reliability assessment result tends security along with increasing the confidence level.

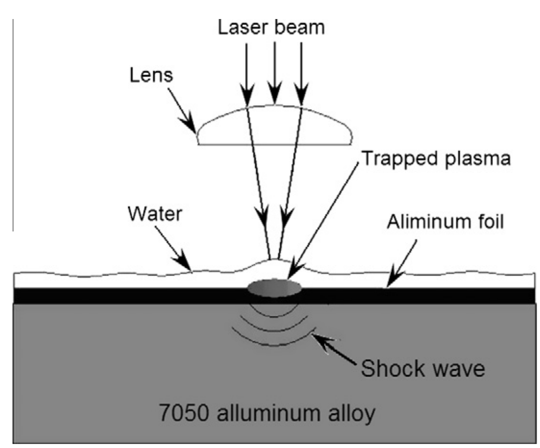

Fig. 1. Laser-shock peening on 7050 aluminum alloy.

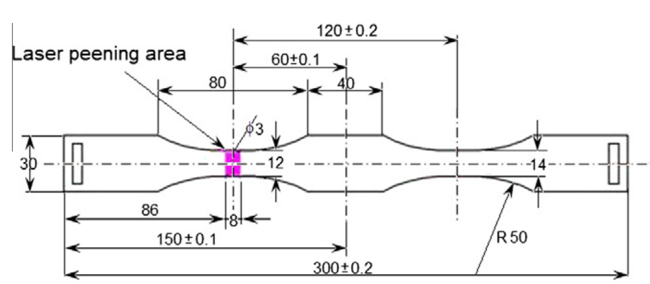

Fig. 2. Test specimen.