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Analytical optimization of constructal channels used for cooling a ring shaped body based on minimum flow and thermal resistances

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ABSTRACT

Constructal theory is invoked to optimize a dendritic path flow structure for minimizing overall flow and thermal resistances. Convective cooling of a ring-shaped body is the aim of this investigation. The construct of flow paths is Y-tree shaped; while the body has heat generation over the area, uniformly. Cooling is done by a single-phase coolant entering through ports located equidistantly along the internal perimeter or enters through the center (when internal radius of convoluted disc-shaped body is zero), and exits through ports located equidistantly along the external perimeter. The regime of fluid flow is laminar and fully developed. The degrees of freedom are the ratio diameters and lengths of ducts. The constraints are the disc size (external radius is fixed) and the total volume of ducts devised in disc. The aim of design is to obtain the best construct, so that two resistances i.e. flow resistance and thermal resistance are minimized. Results show that the mass flow rate has the greatest effect on thermal resistance, but the effect of mass flow rate on flow resistance is negligible. Moreover, it is seen that at high values of pumping power, increasing complexity (more levels of pairing) results in reducing the thermal resistance.

1. Introduction

Applications of constructal theory are visible in nature and engineering [1]. This theory attempts to find the optimal geometries and flow system configurations. One of the resultant configurations of this theory is tree-like architectures. Tree networks can be found in animal flow systems: vessels, lungs, vascularized tissues, etc. [1–4]. Using constructal theory in engineering is observed in fields such as physics, civil engineering, information systems, etc. [5,6].

Tree structures have been proposed by many researchers for cooling the electronic pieces. The cooling of electronics is generally performed by three mechanisms of heat transfer: conduction, convection, and radiation heat transfer. Cooling by conduction is done by embedding high-conductivity inserts in the heat-generating packages [7–12]. For example, the optimization of highly conductive insert architecture was studied for cooling a rectangular chip [9]. More researches in the platform of constructal theory are devoted to convection cooling [13–17]. In this mode of cooling, there exist two goals: optimizing thermal resistance and optimizing flow resistance. Constructal theory obtains optimal geometrical feature of flow paths such that the resistances (thermal resistance and flow resistance) are minimized. This theory was utilized for placement of unequal heat sources on a plate cooled by laminar forced convection [18]. By constructal theory, the effect of svelteness on the bifurcation angles role was studied in pressure drop and flow uniformity of tree-shaped microchannels [19]. The geometric optimization has been studied in different researches. For example, the geometric optimization of an array of circular and non-circular micro-channels was investigated by finite volume method [20].

Lorenzini et al. [21,22] utilized constructal method to find the optimal geometry of X- and Y-shaped cavities intruded into a wall. They further carried out a numerical study to optimize the geometry of a C-shaped cavity intruded into a heat-generating body [23]. Ghaedamini et al. [24] showed that the usage of reverting micro-channels results in a better thermal performance of the disc architectures. Wechsatol et al. [25] optimized a tree shaped design for cooling a disc-shaped body by minimizing the pressure drop between one central point and points located on the perimeter of the disc. They further optimized the previous tree structure used for

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cooling a disc by considering two criteria: minimizing thermal resistance and minimizing distributed pressure drop and observed that these methods lead to nearly the same combined performance [26]. They neglected local pressure drop in their work.

Feng et al. [27] optimized tree-shaped fluid networks in a disc-shaped area by taking total pressure drop as optimization function subjected to total tube surface area constraint. They considered both laminar and turbulent regimes in the tubes. Their results showed that there are optimal angles of tubes which minimize pressure drop. They further found the optimal aspect ratio of the elemental sector in the radial-pattern disc by taking thermal resistance as optimization objective [28]. They also obtained the optimal width ratio of the first-order and elemental cooling channels and the optimal dimensionless radius of the elemental sector.

However, all electronic pieces are not in disc shape. Hence, it is reasonable to examine other popular applied geometries. One of these prevalent geometries is ring shape which allows the wires to pass the electronic piece.

For the first time, Salimpour and Menbari [29] examined the fundamental problem of design and optimization of a tree-shaped flow structure in a ring-shaped heat-generating body by minimizing overall flow resistance. As a continuation of this work, we optimized tree-shaped constructs based on both minimizing flow resistance and minimizing thermal resistance in the present investigation. For this purpose, we considered forced convection cooling of convoluted disc-shaped electronic pieces. We assume the piece to be sufficiently thin that heat generation is distributed over the disc area uniformly. In this study, first we use the results of optimal geometrical features based on minimizing total flow resistance for analyzing the thermal resistance of the system. In the next step, we obtain optimal geometrical features based on minimizing thermal resistance. Afterward, the results of both optimizations are compared, and the best tree-shaped construct which yields the maximum performance of the system is chosen.

2. Modeling of radial architecture

This research aims at designing tree flow paths with the lowest flow resistance (pressure drop) between ports on internal and external perimeters of convoluted-disc body. Tree flow paths are composed of ducts with different diameters (D_i) and various lengths ($L_i, i = 0, 1, 2, \ldots$). The volume occupied by the total ducts is fixed. The regime of fluid flow in different paths is assumed to be laminar and fully developed. The convoluted-disc experiences uniform heat generation per unit area, q''. The single-phase coolant fluid enters the cooling channels via ports located on internal perimeter and exits through ports located on external rim, Fig. 1. The internal radius and external radius of convoluted-disc are r and R, respectively.

Inlet coolant temperature is T_0 and the highest temperature occurred in the two peripheral corners of convoluted-disc is T_m, Fig. 1. The overall maximum temperature different is $T_m - T_0$ and the overall thermal resistance is $(T_m - T_0)/q$ where q is the total heat current generated in the convoluted-disc ($q'' = q/\pi(R^2 - r^2)$). In fact, the thermal resistance is evaluated by noting that the sector is slender enough that the conduction heat transfer through the solid material occurs in the direction perpendicular to the duct. Therefore, by using the first law of thermodynamic for radial duct, we arrive at

$$T_f - T_0 = \frac{q''\pi(R^2 - r^2)}{n\dot{m}_0 c_p} \tag{1}$$

Convection heat transfer equation at the end of duct wall is

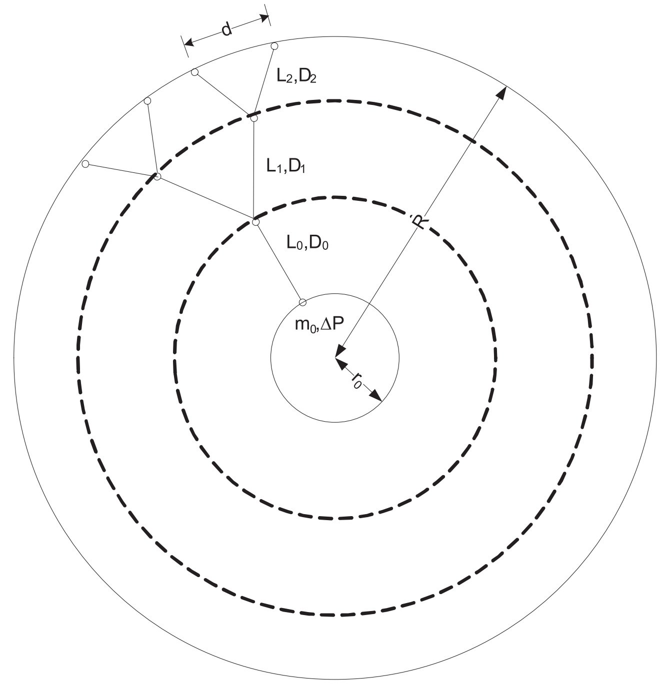

Fig. 1. The tree-shaped construct covering the area of a convoluted disc-shaped body.

$$T_w - T_f = \frac{q''\pi RD}{npNuk_f} \tag{2}$$

Invoking Furrier law for the last ring of disc, we have

$$T_m - T_w = \frac{\pi^2 R^2 q''}{2n^2 kt} \tag{3}$$

where R, r, D, n, $\dot{m}_0$, c_p, Nu, k_f, k and t are external radius, internal radius, diameter of duct, numbers of radial ducts located in convoluted-disc, mass flow rate entering each port of internal perimeter, specific heat at constant pressure, Nusselt number, thermal conductivity of fluid, thermal conductivity of disc and the thickness of disc, respectively. Also, T_f and T_w are the fluid bulk temperature and the duct wall temperature at the duct outlet. In Eq. (3), the heat current arrives at the wall end of the duct is $(q''\pi R)/n$ which is equal to $hp(T_w - T_f)$, where h is the heat transfer coefficient, obtained from $Nu = hD/k_f$. The value p is the wetted perimeter of the duct cross-section. $(T_m - T_w)$ is the temperature distribution on the last ring of disc. The outer part of disc is located along the perimeter of length $d/2$. The heat flux of conduction along the perimeter of this length that arrives at T_m is $kt[4(T_m - T_w)/d]$. By summation of Eqs. (1)–(3), non-dimensional maximum thermal resistance of the convoluted-disc, $\tilde{T}_m = \frac{T_m - T_0}{q/(kt)}$, is obtained.

$$\tilde{T}_m = \frac{1}{M} + \frac{RDkt}{nNuk_f p(R^2 - r^2)} + \frac{\pi R^2}{2n^2(R^2 - r^2)} \tag{4}$$

where, $M = \dot{m}c_p/kt$ is the non-dimensional mass flow rate. Keeping an eye on Eq. (4), it is seen that the non-dimensional maximum thermal resistance constitutes of three terms. The Nusselt number in the fully developed laminar flow is constant, and is of order 1 to 10. The ducts are sufficiently thin, so that the aspect ratio $RD/[p(R^2 - r^2)]$ is smaller than 1. Hence, the second term of Eq. (4) for thin ducts is negligible. The effect of the first term on the non-dimensional maximum thermal resistance is higher than the third

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