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Dynamic penny-shaped cracks in multilayer sandwich composites

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ABSTRACT

A point force method is proposed for obtaining the dynamic elastic response of a multilayer sandwich composite in the presence of a penny-shaped crack under a harmonic loading. The sandwich composite is a multilayered solid whose lower half is the mirror image of the upper half with the center plane as the mirror. The crack is lying on the mirror plane of the composite. The solution of the mode I dynamic crack problem is formulated by integrating the Green's function of a time-harmonic surface normal point force over the crack surface with an unknown point force distribution. The dual integral equations of the unknown point force distribution are established by considering the boundary conditions, which can be reduced to a Fredholm integral equation of the second kind. A complete solution of the crack problem under consideration can be obtained by solving this Fredholm integral equation. It will be shown that the results obtained by this approach are the same as some existing solutions.

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1. Introduction

Interest in crack problems in the mathematical theory of elasticity arises from the study of brittle fracture that originated in the classical work of Griffith [1]. At that time, there were relatively few materials that fail in brittle fashion under normal condition; hence for many years this theory was regarded as academic instead of practical interest. Interest in the theory has been revived in the mid 1960s because of the experimental discovery that at high or low temperatures many structural elements composed of commonly used ductile materials fail by a "quasi-brittle" process. Now, there are a number of books, monographs and journals that devoted to fracture mechanics. Recently, because of advances in computing power, whenever analytical ideas are discussed there is always the point of view that one should just compute the result using some existing software packages. The computational methods in dynamic fractures such as finite element method [2], boundary element method [3,4] and dual boundary element method [5] are very useful for practical geometries. However, except for the question of accuracy there will always be a need for careful analysis to help check and interpret numerical results. This study will provide the analytical solutions of mode I crack problems in multilayer sandwich composites under time-harmonic loadings. Most commonly used sandwich composites such as skies, windshield glasses, vehicle armor panels, and space shuttle heat panels are made by sandwiching two layers of outer plates, called faces, around a different plate, the core, to improve the properties of the materials. The understanding of the dynamic responses of these materials in the presence of cracks will be beneficial to the design and understanding of the properties of these structures.

The effect of time-dependent loading on a stationary crack is of great interests to the fracture mechanics, seismology, geophysics and nondestructive testing and evaluation (NDT/NDE). If the applied loads vary periodically in time, the resulting stress and displacement are propagated through the structure in the form of waves. At a crack, these waves are reflected and refracted causing high local stress intensification at the tips of the crack. The knowledge of reflecting and refracting waves caused by a crack resulting from an externally applied ultrasonic field is also key to the NDT/NDE for detesting and locating cracks in material structures. The diffraction of anti-plane shear waves by a finite crack in an infinite isotropic solid was first studied in [6,7]. The diffraction of a plane longitudinal wave by a penny-shaped crack in an infinite isotropic solid was investigated in [8,9]. In the same time, additional study were made for the normal compression and radial shear waves scattering at a penny-shaped crack in an infinite solid [10] and the torsional wave scattering about a penny-shaped crack lying on a bimaterial interface [11]. The fourth volume of *Mechanics of Fracture* [12] was devoted entirely to dynamically loaded cracks. Since then, there are many studies on the dynamic penny-shaped crack problems [13–20]. All these studies are for cracks either in infinite solids or in semi-infinite solids. For the layered solids, the plane problem of a single crack in a periodically layered bimaterial composite has been considered in [21].

Considered in this study is the axisymmetric problem of determining the steady-state elastic solution of a mode I penny-shaped crack lying on the center layer of a sandwich-layered composite under a time-harmonic loading. In the next section, the basic equations needed for obtaining the solutions of the crack problems

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are given. The method for obtaining the point force solutions (Green's functions) is presented in Section 3. Section 4 is concerned with the stead-state solutions of the crack problems in the multilayer sandwich composites. The solutions are expressed in terms of dual integral equations, which can be reduced to Fredholm integral equations of the second kind. The solutions for a penny-shaped crack in infinite solids, plates, and three-layer sandwich composites are presented as examples. There are many different approaches in the literature to solve the dual integral equations and the Fredholm integral equations of the second kind; the numerical solutions will be report separately.

2. Governing equations

Consider a penny-shaped crack lying on the center (mirror) plane a multilayer sandwich composite, as shown in Fig. 1. The composite consists of $2n + 1$ isotropic homogeneous layers where n is an integer. A cylindrical coordinate system $(r,z) = (r,\theta,z)$ is devised such that the z-axis is normal to the layer interfaces. The lower half $z < 0$ of the composite is the mirror image of the upper half $z > 0$ with $z = 0$ as the mirror plane. Let λ_j and μ_j be the Lame constants, ρ_j and δ_j be the density and thickness of layer j, where $j = -n, -n+1, \ldots, -1, 0, 1, \ldots, n-1, n$. For the sandwich composites, it is assumed that $\lambda_j = \lambda_{-j}$, $\mu_j = \mu_{-j}$, $\rho_j = \rho_{-j}$ and $\delta_j = \delta_{-j}$ ($j \neq 0$). The crack occupies the circle $r = 1$ in the mirror plane $z = 0$ of the core layer $j = 0$.

The surfaces of the crack are stress free and there is a prescribed uniform time-harmonic tensile stress $p_s + p_0e^{i\omega t}$ at the upper and lower free surfaces of the composite. The stress p_s is the static loading, p_0 and ω are the amplitude and frequency of the applied time-harmonic loading, respectively. This is an axisymmetric problem

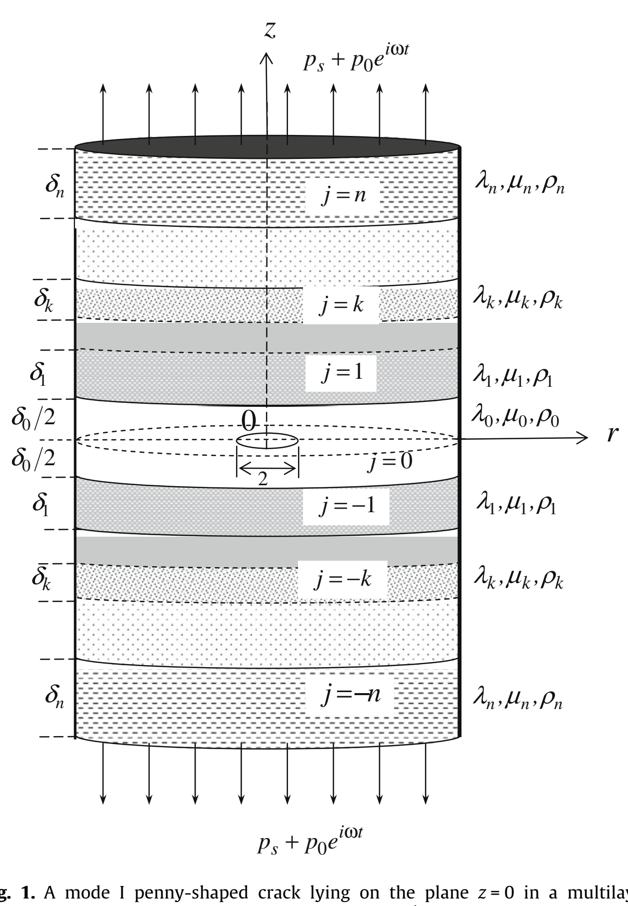

Fig. 1. A mode I penny-shaped crack lying on the plane $z = 0$ in a multilayer sandwich composite under harmonic loading $p_s + p_0e^{i\omega t}$.

that is independent of θ. The boundary conditions at the crack surface are

$$\sigma_z^0 = \sigma_{rz}^0 = 0, \quad (0 \leqslant r \leqslant 1, \quad z = 0), \tag{1}$$

and the boundary conditions on the upper and lower facing surfaces are

$$\sigma_z^{\pm n} = p_s + p_0e^{i\omega t}, \quad \sigma_{rz}^{\pm n} = 0, \quad (z = \pm d, \quad -\infty < r < \infty), \tag{2}$$

where

$$d = \frac{\delta_0}{2} + \sum_{i=1}^{n} \delta_i, \tag{3}$$

σ_z^j and σ_{rz}^j are the normal and shear stresses in layer j. The r- and z-components of displacement in the layer j will be denoted by u_r^j and u_z^j, respectively. The apply static tension p_s is assumed large enough to ensure that the crack faces do not make contact during vibration. Because of the symmetry in the properties and thicknesses of each layer in the sandwich-layered composites, the distribution of displacements and stresses resulting from the crack is the same as that produced in a composite $0 \leqslant z \leqslant d$ (Fig. 2), when its free surface, $z = 0$, is subject to the following boundary conditions [8,22,23]

$$\sigma_{rz}^0 = 0, \quad (0 \leqslant r < \infty), \tag{4a}$$

$$\sigma_z^0 = -p_s - p_0e^{i\omega t}, \quad (0 \leqslant r < 1), \tag{4b}$$

$$u_z^0 = 0, \quad (r > 1). \tag{4c}$$

Since solution of elastic problems may be superimposed, condition (4b) will be written in the meantime as

$$\sigma_z^0 = -p_0e^{i\omega t}, \quad (0 \leqslant r < 1). \tag{4d}$$

The solutions of the static problems with

$$\sigma_z^0 = -p_s, \quad (0 \leqslant r < 1), \tag{4e}$$

will be considered in Appendix B. The outer surface $z = d$ is a free surface where

$$\sigma_z^n = \sigma_{rz}^n = 0, \quad (0 \leqslant r < \infty). \tag{5}$$

When the layers are perfectly bonded to each other, the boundary conditions at the interfaces $z = z_i$, $-\infty \leqslant r \leqslant \infty$ between layer j and layer $j + 1$ are

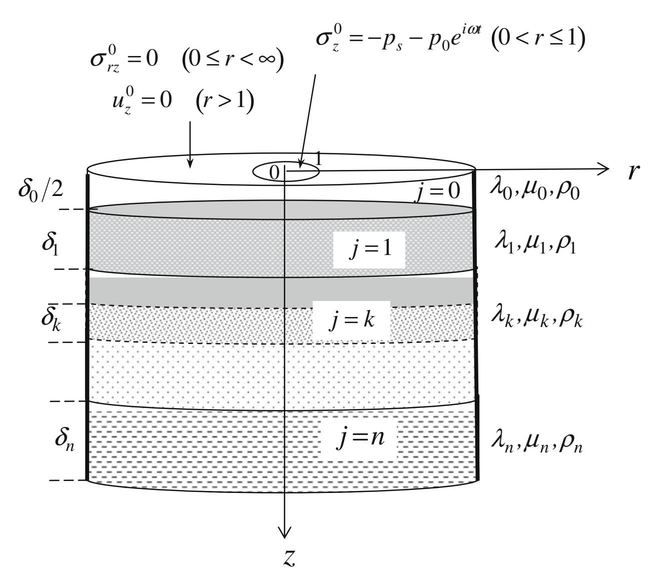

Fig. 2. The boundary value problem for the half multilayer sandwich composite $z \geqslant 0$ corresponding to Mode I displacement in a penny-shaped crack shown in Fig. 1.