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Crack movement in layered composites under electric field affect

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ABSTRACT

The variation principle is applied for defining a crack in the solid body. The methods proposed in [G. Sih, C. Chen, Non-self-similar crack growth in elastic–plastic finite thickness plate, Theoretical and Applied Fracture Mechanics 3 (1985) 125–139] extend to presence of electromagnetic fields in material. Crack propagation in non-homogeneous media has been considered. It is shown that electromagnetic fields in the material are essentially affecting the trajectory. The crack trajectory stability has been studied as function of fracture energy, phase portraits of the trajectory in different media have been built, and various attractor types have been revealed. Different crack morphologies from single straight and oscillating crack propagation to straight double crack propagation were theoretically founded. In compliance with the experimental data of [R. Niefanger, V.-B. Pham, G. Schneider, H.-A. Bahr, H. Balke, U. Bahr, Quasi-static straight and oscillatory crack propagation in ferroelectric ceramics due to moving electric field: experiments and theory, Acta Materialia 52 (1) (2004) 117–127], it has been demonstrated that periodic electromagnetic field results in trajectory stochastization. This can be used for switching the crack over from the mode of mainline propagation into the mode of development of the field of diffused microcracks.

1. Introduction

The issue of a fracture front shape is crucial for studying the behavior of materials and structures in critical operation conditions, close to the area of potential destructions. As a rule, the fracture is treated as a mathematical cut that propagates rectilinearly in some homogeneous (ideal) material. However, a deeper analysis shows that a correct application of linear destruction mechanics results in demonstration of the fact that the fracture front is curvilinear. It was shown in [1], which for some reasons failed to attract due and thorough attention of researchers working in the field. The issue of fracture trajectory in a non-uniform ambient (for example, in a material with varying mechanical constants) or in a medium full of non-uniform electromagnetic field, has been insufficiently studied.

Piezoelectric and ferroelectric ceramics are attracting additional attention, since they are more and more broadly used recently. Simultaneously the some properties of the fracture not investigated satisfactory till now. An influence of the electric field upon the fracture toughness has been observed by many researchers experimentally [2] and theoretically. One of the first fundamental works in the field of piezomaterial fracture [3] has formulated the basic problems, various aspects of which, especially mathematic ones, are intensely studied [4–9]. Possibility to apply different fracture criterion to piezomaterial discussed in [10].

The work below is a further development of the method that can be used for predicting a fracture trajectory in non-uniform media [11], in case the material has electromagnetic field inside.

2. Crack energy density criterion and trajectory of crack

2.1. Crack trajectory: problem statement

It is widely known that the problem of determining the extreme of any functional correspond to solving the Euler–Lagrange equation. For example, the main idea of the Sih criterion of fracture [12] for determining the direction of crack propagation is to find the minimum of the strain energy density function. Mathematically, this problem can be reduced to the variational problem for the potential function because the strain energy density function is the part of total potential energy of the body. Geometrically speaking, the solution of the Euler–Lagrange equation is the equation of the geodesic line [13]. A crack's trajectory obtained in this way is a geodesic line in a specially constructed state space. Apparently, such a view was also considered by [14]. Let the medium be elastic and is governed by the relation

$$\varepsilon_{ij}(x,y) = C_{ijkl}\sigma_{kl}(x,y) \tag{1}$$

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where $\varepsilon_{ij}(x,y)$ is the strain tensor, C_{ijkl} is the compliance coefficients tensor and $\sigma_{kl}(x,y)$ is the stress tensor. Consider the medium that is stretched along the y-axis from infinity. The equation for specific energy (i.e. energy per unit length) has the form [12,15]

$$\delta\int_{\varepsilon_B}^{\varepsilon_A}(2\gamma - P_1u_1)ds = 0, \quad (2\gamma - P_1u_1) = F(x,y,\dot{y}) \tag{2}$$

where γ is the density of surface energy of the material, $P_i = \sigma_{ij}n_j$ are the stress tensor components on the pads, the position of which coincides with the crack surface, n_i is the guiding cosine of the external normal to the crack surface, u_i is the displacement of the crack edges, $\varepsilon_A, \varepsilon_B$ are the – initial and final crack lengths, accordingly. The equation of crack trajectory is written down in the form as: $y = y(x)$.

Leaving out the fractal properties of the trajectory and suppose that trajectory is sufficiently smooth, it is possible to reduce Euler equation to form [11,13]

$$y'' - y'f_1(x,y) + f_2(x,y) = 0 \tag{3}$$

where the notations

$$f_1(x,y) = \frac{\partial \ln Q(x,y)}{\partial x}, \quad f_2(x,y) = \frac{\partial \ln Q(x,y)}{\partial y},$$
$$Q(x,y) = F^{-1}(x,y) \tag{4}$$

are have been adopted.

2.2. The trajectory in layered media

Consider a crack trajectory in periodically non-homogeneous medium, defined by the correlation of the type:

$$(\ln Q(x,y))_{,x} = \delta + \gamma\cos\omega x \tag{5}$$

By integrating (5), we can obtain a clear expression for function $Q(x) = F(x)^{-1}$:

$$Q(x) = C_1\exp\left(-\frac{\delta x\omega + \gamma\sin(\omega x)}{\omega}\right) \tag{6}$$

where C_1 is the integration constant. The behavior of the non-homogeneity is in general exponential (the slope degree of the exponent is regulated by index δ). A deviation of the function from smoothness (scatter of the properties of the composite by layers) is regulated by parameter γ.

By substituting (5) into (3), the Duffing equation is obtained, containing $\dot{y}$, as equation of the mid-plane line shape of crack:

$$\begin{aligned} &\ddot{y} - y + y^3 + \delta\dot{y} = \gamma\dot{y}\cos\omega x \\ &-f^0_{2,y} = +1, -\tfrac{1}{2}f^0_{2,y^2} = 0, -\tfrac{1}{3}f^0_{2,y^3} = -1 \end{aligned} \tag{7}$$

It has been shown in [16] that in this case the system is not the Hamiltonian one, and it is necessary to find the conditions for the onset of deterministic chaos. In a similar way [17], the conditions for transition from the deterministic motion of the beam trajectory to the chaotic one are found.

According to the representation of Eq. (7), the crack trajectory depends on the composite constituents described by the parameters γ, ω and δ.

The results of building phase portraits and the trajectory of crack propagation for different initial conditions are presented in Figs. 1–8. The equation was integrated by the standard Runge–Kutta method with degree four interpolation. The integration step was defined by the capacities of the used computers and in any case did not exceed 0.05. It follows from the curves on the Fig. 2, the system has a very fast stabilization. The phase portrait contains the stable focus. It means that fluctuation in the media parameters is small relative of medium properties and instability not growth.

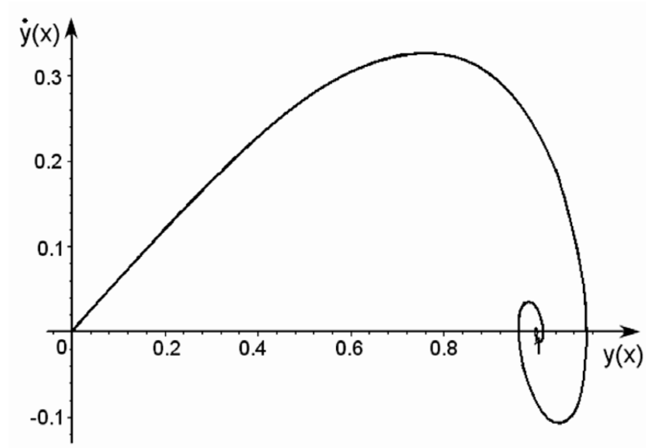

Fig. 1. Phase portrait of crack trajectory. Values $\gamma = 0.1$; $\omega = 0.1$; $\delta = 1$; initial conditions $y(0) = 0.0001$, $\dot{y}(0) = 0.005$.

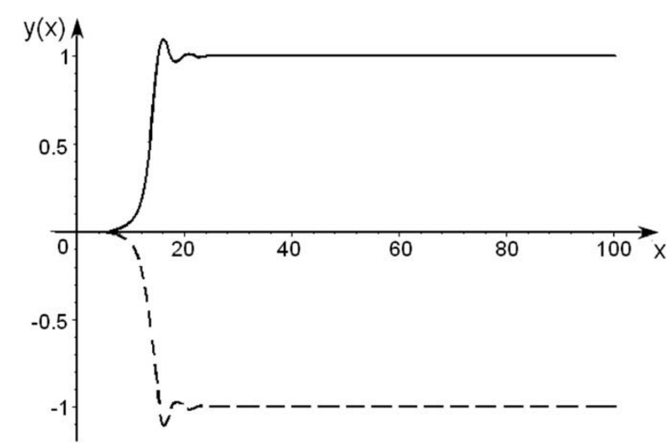

Fig. 2. Crack trajectory. Initial conditions $y(0) = 0$, $\dot{y}(0) = 0.0005$ – solid line, $y(0) = 0$, $\dot{y}(0) = -0.0005$ dotted line.

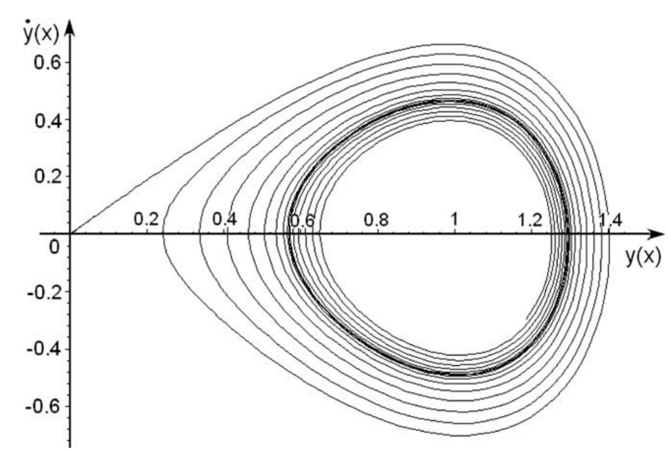

Fig. 3. Phase portrait. Values $\gamma = 0.01$; $\omega = 0.1$; $\delta = 0.1$.

Figs. 2 and 4 illustrate the dependence of the behavior of the trajectory on parameter δ. It is seen that the trajectories are similar in the principle.

The case Figs. 3 and 4 the weak attractor exists but it is broken under the weak excitation. In case Fig. 6 the crack trajectory is unstable, but standard Maple computing algorithm limited possible diapason of investigation.