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Cracking characteristics of mixed mode dislocations near a lip-like mode crack

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ABSTRACT

This work is concerned with the cracking characteristics of mixed mode dislocations near a lip-like mode crack, stress intensity and strain energy density factor are obtained by using conformal mapping, singularity analysis and Cauchy integrals. Shielding effect generated by screw dislocation near a lip-like mode crack decreases with the increment of the distance between screw dislocation and crack tip. Larger distance between two faces of the crack leads to the shielding effect waning. The strain energy density factor of mode III decreases with the increment of the distance between dislocation and crack tip. Larger distance between two faces of lip-like mode crack also leads to the strain energy density factor waning and encourages crack initiation; the shielding effects generated by edge dislocation near the crack decrease with the increment of the distance between edge dislocation and crack tip.

1. Introduction

The study of dislocations interacting with cracks in the mechanics and materials science is motivated by the need of a better understanding of the mechanism of strengthening and toughening of materials. The stress intensity factor and strain energy density factor are the major concerns in the investigations of cracking characteristics. In last several decades, many researchers discussed these problems, the concepts of stress intensity factor criterion and strain energy density factor criterion were given by [1–3]. The stress intensity factor of a screw dislocation near a semi-infinite crack was obtained in [4]. Presented in [5] are some general solutions for crack tip shielding and anti-shielding by screw and edge dislocations. The initiation and growth characterization of corner cracks near circular hole was researched in [6]. Recently, both of the criterions are applied to composites with interfacial crack and dislocations. Cracking characteristics of a moving screw dislocation near an interfacial crack in two dissimilar orthotropic media has been discussed in [7,8] study a technique for studying interaction between a screw dislocation dipole or a concentrated load and a mode III crack crossing an interface. Screw dislocation interacting with interfacial and interface cracks in piezoelectric bi-materials has been investigated in [9].

Nevertheless, the previous formulations assume the crack to be a line. There is distance between two faces of real crack, the distance has important influences on the stress intensity factor and strain energy density factor. The lip-like mode crack is more practice than Griffith crack and more ease than elliptically blunted crack which discussed in [10] in engineering fracture.

In this paper, the influence of the distance between two faces of crack and the position of dislocations on shielding effect and strain energy density factor has been investigated by using complex mapping, the technique of singularity analysis of complex functions and Cauchy integral. The presented solutions contain previously known results as the special cases.

2. Problem description

As shown in Fig. 1, a lip-like mode crack locates on the x axis of the infinite elastic plane with length of $2l$ and distance of $2h$. The end points of crack are two sharp point l and $-l$. Screw dislocation and edge dislocation locate at the point z_0 near the crack tip, the dislocations are characterized by Burgers vectors b_z and b_x, b_y, respectively.

By using Mapping function

$$z = \frac{l}{2}(t + t^{-1}) \tag{1}$$

The lip-like mode crack in complex z-plane is mapped into a tangent ellipse of unit circle in t-plane, as shown in Fig. 2.

By using Mapping function

$$t = i\rho(\zeta + m\zeta^{-1}) \tag{2}$$

The tangent ellipse of unit circle in t-plane is mapped into a unit circle in ζ-plane, as shown in Fig. 3.

Substituting Eq. (2) into Eq. (1), yield the mapping function

$$z = \omega(\zeta) = \frac{l\rho i}{2}\left[\zeta + \frac{m}{\zeta} - \frac{\zeta}{\rho^2(\zeta^2 + m)}\right] \tag{3}$$

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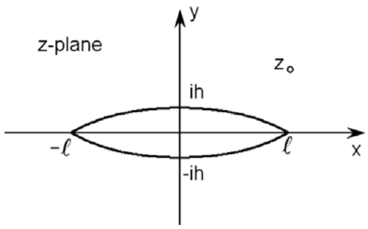

Fig. 1. z-Plane.

where $\rho = \frac{a+1}{2}$, $m = \frac{a-1}{a+1}$, $a = \frac{h}{l} + \sqrt{\frac{h^2}{l^2} + 1}$, $\zeta = re^{i\varphi}$.

The lip-like mode crack in complex z-plane is mapped into an unit circle in ζ-plane.

3. The solution of the problem

3.1. Screw dislocation near the lip-like mode crack tip

Letting displacement on z-plane

$$w = \mathrm{Re} f(z) \tag{4}$$

where "Re" denotes obtaining the real component.

Referring to the work of [11], the analytic function $f(z)$ of the infinite material which includes crack and a screw dislocation can be written as

$$f(z) = \frac{b_z}{2\pi i} \ln(z - z_0) + f_0(z) \quad z \in S^- \tag{5}$$

Stress on z-plane can be written as Cartesian coordinates form

$$\tau_{xz} - i\tau_{yz} = \mu f'(z) \tag{6}$$

Pole coordinates form

$$\tau_{rz} - i\tau_{\theta z} = e^{i\theta} \mu f'(z) \tag{7}$$

Substituting Eq. (3) into Eqs. (5) and (7), the analytic function $f(\zeta)$ and stress components can be written as , respectively. Analytical function

$$f(\zeta) = \frac{b_z}{2\pi i} [\ln(\zeta - \zeta_0) + \ln(\zeta - \zeta_1) + \ln(\zeta - \zeta_2) + \ln(\zeta - \zeta_3) - \ln(\zeta - \zeta_4) - \ln(\zeta - \zeta_5) - \ln \zeta] + f_0(\zeta) \quad \zeta \in S^- \tag{8}$$

where $\zeta_1 = m/\zeta_0$, $\zeta_2 = \frac{-\chi + \sqrt{\chi^2 - 4m}}{2}$, $\zeta_3 = \frac{-\chi - \sqrt{\chi^2 - 4m}}{2}$, $\chi = \frac{1}{\rho^2 \frac{\zeta_0^2 + m}{\zeta_0}} \zeta_4 = i\sqrt{m}$, $\zeta_5 = -i\sqrt{m}$, $|\zeta_i| < 1$ ($i = 1, 2, 3, 4, 5$) stress components

$$\tau_{rz} - i\tau_{\theta z} = \frac{\mu \zeta f'(\zeta)}{|\omega'(\zeta)| r} \tag{9}$$

The first order differential form of $f(\zeta)$ is shown as

$$f'(\zeta) = \frac{b_z}{2\pi i} \left[\frac{1}{\zeta - \zeta_0} + \frac{1}{\zeta - \zeta_1} + \frac{1}{\zeta - \zeta_2} + \frac{1}{\zeta - \zeta_3} - \frac{1}{\zeta - \zeta_4} - \frac{1}{\zeta - \zeta_5} - \frac{1}{\zeta} \right] + f_0'(\zeta) \quad \zeta \in S^- \tag{10}$$

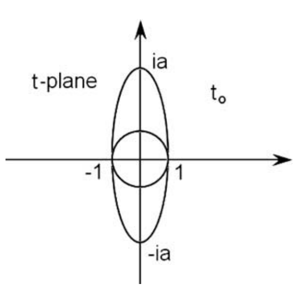

Fig. 2. t-Plane.

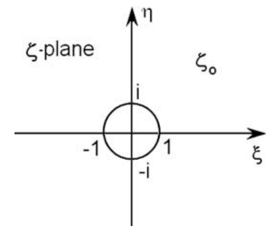

Fig. 3. ζ-Plane.

where $f_0'(\zeta) = O\left(\frac{1}{\zeta}\right)$ denotes high order of $\frac{1}{\zeta}$.

Defining a new analytic function

$$f_*'(\zeta) = -\frac{1}{\zeta^2} \overline{f'}\left(\frac{1}{\zeta}\right) \quad \zeta \in S^+ \tag{11}$$

Substituting Eq. (10) into Eq. (11), yield

$$f_*'(\zeta) = \frac{-b_z}{2\pi i} \left[\frac{1}{\zeta - 1/\bar{\zeta}_0} + \frac{1}{\zeta - 1/\bar{\zeta}_1} + \frac{1}{\zeta - 1/\bar{\zeta}_2} + \frac{1}{\zeta - 1/\bar{\zeta}_3} - \frac{1}{\zeta - 1/\bar{\zeta}_4} - \frac{1}{\zeta - 1/\bar{\zeta}_5} - \frac{1}{\zeta} \right] + f_{0*}'(\zeta) \quad \zeta \in S^+ \tag{12}$$

The boundary condition of crack surface

$$\tau_{rz}(n) = 0 \tag{13}$$

where $n = e^{i\varphi}$ locates on the unit circle of ζ-plane.

Noting Eq. (9), yield

$$nf'(n) + \overline{nf'(n)} = 0 \tag{14}$$

In view of Eq. (11), yield

$$nf'(n) + \overline{nf'(n)} = nf'(n) + \frac{1}{n} \frac{f_*'(n)}{-\frac{1}{n^2}} = nf'(n) - nf_*'(n) = 0 \tag{15}$$

namely

$$f_*'^{+}(n) - f'^{-}(n) = 0 \tag{16}$$

Substituting Eqs. (10) and (12) into Eq. (16), the boundary problem can be written as

$$f_{0*}'^{+}(n) - f_0'^{-}(n) = A(n) \tag{17}$$

where

$$A(n) = \frac{b_z}{2\pi i} \left[\frac{1}{n - 1/\bar{\zeta}_0} + \frac{1}{n - 1/\bar{\zeta}_1} + \frac{1}{n - 1/\bar{\zeta}_2} + \frac{1}{n - 1/\bar{\zeta}_3} - \frac{1}{n - 1/\bar{\zeta}_4} - \frac{1}{n - 1/\bar{\zeta}_5} - \frac{1}{n} \right] + \frac{b}{2\pi i} \left[\frac{1}{n - \zeta_0} + \frac{1}{n - \zeta_1} + \frac{1}{n - \zeta_2} + \frac{1}{n - \zeta_3} - \frac{1}{n - \zeta_4} - \frac{1}{n - \zeta_5} - \frac{1}{n} \right] \tag{18}$$

Using Plemej equation, yield

$$f_{0*}'(\zeta) = \oint_{|\zeta|=1} \frac{A(n)}{n - \zeta} dn + c\zeta \in S^+ \tag{19}$$

$$f_0'(\zeta) = \oint_{|\zeta|=1} \frac{A(n)}{n - \zeta} dn + c\zeta \in S^- \tag{20}$$

Using Cauchy integral, yield

$$f_0'(\zeta) = \frac{b_z}{2\pi i} \left[-\frac{1}{\zeta - 1/\bar{\zeta}_0} + \frac{1}{\zeta} + \frac{1}{\zeta - \zeta_0} - \frac{1}{\zeta - \zeta_1} - \frac{1}{\zeta - \zeta_2} - \frac{1}{\zeta - \zeta_3} + \frac{1}{\zeta - \zeta_4} + \frac{1}{\zeta - \zeta_5} + \frac{1}{\zeta} \right] + c \tag{21}$$

In view of the value of $f_0'(\zeta)$ at infinite, yield $c = 0$.

Substituting Eq. (21) into Eq. (10), yield